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Flexural Motion Under Moving Masses Of Prestressed Simply Supported Plate Resting On Bi-parametric Foundation

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A B S T R A C T

In this investigation, the flexural vibration of a prestressed and simply supported rectangular plate carrying moving concentrated masses and resting on bi-parametric (Pasternak) elastic foundation is considered. In order to solve the governing fourth order partial differential equation, a technique based on separation of variables is used to reduce the equation with variable and singular coefficients to a sequence of coupled second order ordinary differential equations. The modified method of Struble and the integral transformations are then employed for the solutions of the reduced equations. The numerical results in plotted curves show that as the value of the axial force in x-direction (N_x) increases, the response amplitudes of the plates decrease, the same effect is produced as the axial force in y-direction (N_y) increases for both cases of moving force and moving mass problems of the prestressed and simply supported rectangular plate resting on Pasternak elastic foundation. The deflection of the plate also decreases in each case as the values of the Shear modulus G_o and the rotatory inertia correction factor R_o increase. Also, the transverse deflections of the prestressed rectangular plates under the actions of moving masses are higher than those obtained when only the force effects of the moving loads are considered. Further analysis shows that resonance is attained earlier in moving mass problem than in moving force problem and that the critical speed for the moving mass problem is reached prior to that of the moving force problem, implying that it is risky to rely on a design based on the moving force solution. Furthermore, the response amplitudes of the moving mass problem increase with increasing mass ratio and approach those of the moving force as the mass ratio approaches zero for the prestressed and simply supported rectangular plates resting on uniform Pasternak elastic foundation.

Key words:
Prestress,
Rotatory Inertia,
Bi-parametric Foundation,
Resonance,
Critical speed,
Mass Ratio.

1. Introduction

The behaviour of plate structures under moving load, in general, is rather complex, more so when the inertia effect of the moving load is taken into consideration (Fryba 1972). Thus, most of the research works available in literature are those in which this effect has been neglected. This is due, at least in part, to the great amount of computational labour, which is required both to set up and to solve the necessary equations. One important problem that arises when the inertia effects of the masses are considered is the singularity which occurs in the inertia terms of the governing differential equation of motion.

Many researchers have made tremendous efforts in analyzing the dynamic response of elastic structures under the action of moving masses (Oni 1991, Inglis 1934, Gbadeyan and Aiyesimi 1990, Sadiku and Leipholz 1981, Gbadeyan and Oni 1995). In most analytical studies in Engineering and Mathematical Physics, structural members are commonly modeled as a beam or as a plate. Generally, the dynamical problems of structures under moving loads and resting on a foundation are complex. Earlier

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researchers into beam member on elastic foundation include Franklin and Scott (1979) and Lentini (1979) who presented a finite difference method to solve the problem. These works, though useful, considered the loads acting on the beams to be static. Recently, Oni and Awodola (2005) extended the works of these previous authors to investigate the dynamic response to moving concentrated masses of uniform Rayleigh beams resting on Winkler elastic foundation. Oni and Awodola (2010) again considered the dynamic response under a moving load of an elastically supported non-prismatic Bernoulli-Euler beam. The technique was based on the generalized Galerkin's method and integral transformations.

The foundation model based on Winkler's approximation model is very common in literature, whereas, in such an important Engineering problem as the vibration of plates resting on elastic foundation, a more accurate Two-Parameter (Pasternak) foundation model which in addition to subgrade modulus incorporates the shear effect of the foundation should be used rather than the Winkler's approximation model. Eisenberger and Clastornik (1987) presented two methods for the solution of beams on two-parameter elastic foundation. Also, Gbadeyan and Oni (1992) studied the dynamic analysis of an elastic plate continuously supported by an elastic Pasternak foundation traversed by an arbitrary number of concentrated masses.

Also, Engineers often create artificial stresses in structures before loading; such artificial stresses are forces which may act axially or otherwise. When they act axially, they are called axial forces. The artificial stresses are also called prestress. The aim of prestress is to limit tensile stresses and hence flexural cracking or bending in the structure under working conditions. If the structure is subjected to a force parallel to its axes in addition to the lateral loading, the local equilibrium of forces is altered because the axial force interacts with the lateral displacement to produce an additional term (Clough and Penziens 1975). This additional term due to the axial force increases the complexity of the problem.

In all these investigations, extension of the theory to cover two-dimensional (plate) problem in which the plate is prestressed and rests on bi-parametric foundation has not been effected. This study is therefore concerned with the behaviour of prestressed simply supported rectangular plate under the action of concentrated moving masses and resting on uniform bi-parametric (Pasternak) elastic foundation.

2. Governing Equation

The dynamic transverse displacement H(x,y,t) of a rectangular plate when it is resting on a constant Pasternak elastic foundation and traversed by concentrated masses M_i moving with velocity c_i is governed by the fourth order partial differential equation given by Oni and Awodola (2010)

$$D\nabla^4 H(x,y,t) + \mu\frac{\partial^2 H(x,y,t)}{\partial t^2} = \mu R_0\left[\frac{\partial^4}{\partial t^2 \partial x^2} + \frac{\partial^4}{\partial t^2 \partial y^2}\right]H(x,y,t) + \left[N_x\frac{\partial^2 H(x,y,t)}{\partial x^2} + N_y\frac{\partial^2 H(x,y,t)}{\partial y^2}\right.$$
$$- F_0 H(x,y,t) + G_0\left[\frac{\partial^2 H(x,y,t)}{\partial x^2} + \frac{\partial^2 H(x,y,t)}{\partial y^2}\right] + \sum_{i=1}^{N}\left[M_i g\delta(x-c_i t)\delta(y-s)\right.$$
$$\left. - M_i\left(\frac{\partial^2}{\partial t^2} + 2c_i\frac{\partial^2}{\partial t\partial x} + c_i^2\frac{\partial^2}{\partial x^2}\right)H(x,y,t)\delta(x-c_i t)\delta(y-s)\right] \quad (1$$

where $$D = \frac{Eh^2}{12(1-v)} \quad (2)$$

is the bending rigidity of the plate, 2Ñ is the two-dimensional Laplacian operator, h is the plate's thickness, E is the Young's Modulus, v is the Poisson's ratio, m is the mass per unit area of the plate, 0R is the Rotatory inertia correction factor, F_0 is the foundation constant, G_0 is the constant shear modulus, g is the acceleration due to gravity, δ(.) is the Dirac-Delta function, N_x and N_y are the axial forces in x and y directions respectively, x and y are respectively the spatial coordinates in x and y directions and t is the time coordinate.

The initial conditions, without any loss of generality, is taken as

$$H(x,y,t) = 0 = \frac{\partial H(x,y,t)}{\partial t} \quad (3a)$$

For the elastic rectangular plate resting on Pasternak elastic foundation and having simple supports at all its edges, the deflection and bending moment vanish at all the edges. Thus

$$H(0,y,t)=0, \qquad H(L_X,y,t)=0 \tag{3b}$$

$$H(x,0,t)=0, \qquad H(x,L_Y,t)=0 \tag{3c}$$

$$\frac{\partial^2 H(0,y,t)}{\partial x^2}=0, \qquad \frac{\partial^2 H(L_X,y,t)}{\partial x^2}=0 \tag{3d}$$

$$\frac{\partial^2 H(x,0,t)}{\partial y^2}=0, \qquad \frac{\partial^2 H(x,L_Y,t)}{\partial y^2}=0 \tag{3e}$$

and for the normal modes

$$\Psi_{ni}(0)=0, \qquad \Psi_{ni}(L_X)=0 \tag{3f}$$

$$\Psi_{nj}(0)=0, \qquad \Psi_{nj}(L_Y)=0 \tag{3g}$$

$$\frac{\partial^2 \Psi_{ni}(0)}{\partial x^2}=0, \qquad \frac{\partial^2 \Psi_{ni}(L_X)}{\partial x^2}=0 \tag{3h}$$

$$\frac{\partial^2 \Psi_{nj}(0)}{\partial y^2}=0, \qquad \frac{\partial^2 \Psi_{nj}(L_Y)}{\partial y^2}=0 \tag{3i}$$

3. Analytical Approximate Solution

This section seeks to obtain the analytical solution to the problem of the dynamic response of a prestressed and simply supported rectangular plate resting on Pasternak elastic foundation. The method of analysis involves expressing the Dirac – Delta function as a Fourier cosine series. A technique (Shadnam et al, 2001) based on separation of variables is used to reduce the fourth order partial differential equation governing the motion of the plate to a set of coupled second order ordinary differential equations. The modified asymptotic method of Struble in conjunction with the techniques of integral transformation and convolution theory are then employed to obtain the closed form solution of the resulting second order ordinary differential equations. In order to solve equation (1), in the first instance, the deflection is written in the form (Shadnam et al, 2001)

$$H(x,y,t)=\sum_{n=1}^{\infty}\phi_n(x,y)U_n(t) \tag{4}$$

where ϕ_n are the known eigenfunctions of the plate with the same boundary conditions. The $_n$ have the form of

$$\nabla^4\phi_n-\omega_n^4\phi_n=0 \tag{5}$$

$$\text{where } \omega_n^4=\frac{\Omega_n^2\mu}{D} \tag{6}$$

Ω_n, n = 1, 2, 3, … are the natural frequencies of the dynamical system and Un(t) are amplitude functions which have to be calculated.

In order to solve equation (1), it is rewritten as

$$\frac{D}{\mu}\nabla^4 H(x,y,t)+\frac{\partial^2 H(x,y,t)}{\partial t^2}=R_0\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]H(x,y,t)$$
$$+\left[\frac{N_x}{\mu}\frac{\partial^2 H(x,y,t)}{\partial x^2}+\frac{N_y}{\mu}\frac{\partial^2 H(x,y,t)}{\partial y^2}\right]-\frac{F_0}{\mu}H(x,y,t)+\frac{G_0}{\mu}\left[\frac{\partial^2 H(x,y,t)}{\partial x^2}+\frac{\partial^2 H(x,y,t)}{\partial y^2}\right]$$
$$+\sum_{i=1}^{N}\left[\frac{M_i g}{\mu}\delta(x-c_it)\delta(y-s)-\frac{M_i}{\mu}\left(\frac{\partial^2}{\partial t^2}+2c_i\frac{\partial^2}{\partial t\partial x}+c_i^2\frac{\partial^2}{\partial x^2}\right)H(x,y,t)\delta(x-c_it)\delta(y-s)\right] \tag{7}$$

The right hand side of equation (7) is written in the form of a series to have

$$R_0\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]H(x,y,t)+\left[\frac{N_x}{\mu}\frac{\partial^2 H(x,y,t)}{\partial x^2}+\frac{N_y}{\mu}\frac{\partial^2 H(x,y,t)}{\partial y^2}\right]$$
$$-\frac{F_0}{\mu}H(x,y,t)+\frac{G_0}{\mu}\left[\frac{\partial^2 H(x,y,t)}{\partial x^2}+\frac{\partial^2 H(x,y,t)}{\partial y^2}\right]+\sum_{i=1}^{N}\left[\frac{M_i g}{\mu}\delta(x-c_it)\delta(y-s)\right.$$
$$\left.-\frac{M_i}{\mu}\left(\frac{\partial^2}{\partial t^2}+2c_i\frac{\partial^2}{\partial t\partial x}+c_i^2\frac{\partial^2}{\partial x^2}\right)H(x,y,t)\delta(x-c_it)\delta(y-s)\right]=\sum_{n=1}^{\infty}\phi_n(x,y)\theta_n(t) \tag{8}$$

Substituting equation (4) into equation (8) we have

$$\sum_{n=1}^{\infty}\left\{R_0\left[\phi_{n,xx}(x,y)U_{n,tt}(t)+\phi_{n,yy}(x,y)U_{n,tt}(t)\right]-\frac{F_0}{\mu}\phi_n(x,y)U_n(t)+\frac{N_x}{\mu}\phi_{n,xx}(x,y)U_n(t)\right.$$
$$+\frac{N_y}{\mu}\phi_{n,yy}(x,y)U_n(t)+\frac{G_0}{\mu}\left[\left(\phi_{n,xx}(x,y)U_n(t)+\phi_{n,yy}(x,y)U_n(t)\right)\right]+\sum_{i=1}^{N}\left[\frac{M_i g}{\mu}\delta(x-c_it)\delta(y-s)\right.$$
$$\left.\left.-\frac{M_i}{\mu}\left(\phi_n(x,y)U_{n,tt}(t)+2c_i\phi_{n,x}(x,y)U_{n,t}(t)+c_i^2\phi_{n,xx}(x,y)U_n(t)\right)\delta(x-c_it)\delta(y-s)\right]\right\}=\sum_{n=1}^{\infty}\phi_n(x,y)\theta_n(t) \tag{9}$$

where

$$\phi_{n,x}(x,y)\ implies\ \frac{\partial\phi_n(x,y)}{\partial x},\quad \phi_{n,xx}(x,y)\ implies\ \frac{\partial^2\phi_n(x,y)}{\partial x^2},$$
$$\phi_{n,y}(x,y)\ implies\ \frac{\partial\phi_n(x,y)}{\partial y},\quad \phi_{n,yy}(x,y)\ implies\ \frac{\partial^2\phi_n(x,y)}{\partial y^2}, \tag{10}$$
$$U_{n,t}(t)\ implies\ \frac{dU_n(t)}{dt}\ and\ U_{n,tt}(t)\ implies\ \frac{d^2U_n(t)}{dt^2}$$

Multiplying both sides of equation (9) by $\phi_p(x,y)$ and integrating on area A of the plate, we have

$$\sum_{n=1}^{\infty}\int_A \Big\{R_0\big[\phi_{n,xx}(x,y)\phi_p(x,y)U_{n,tt}(t)+\phi_{n,yy}(x,y)\phi_p(x,y)U_{n,tt}(t)\big]+\frac{N_x}{\mu}\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)$$
$$+\frac{N_y}{\mu}\phi_{n,yy}(x,y)\phi_p(x,y)U_n(t)-\frac{F_0}{\mu}\phi_n(x,y)\phi_p(x,y)U_n(t)+\frac{G_0}{\mu}\big[\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)+\phi_{n,yy}(x,y)\phi_p(x,y)U$$
$$+\sum_{i=1}^{N}\Big[\frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_it)\delta(y-s)-\frac{M_i}{\mu}\Big(\phi_n(x,y)\phi_p(x,y)U_{n,tt}(t)+2c_i\phi_{n,x}(x,y)\phi_p(x,y)U_{n,t}(t)$$
$$+c_i^2\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)\Big)\delta(x-c_it)\delta(y-s)\Big]\Big\}dA=\sum_{n=1}^{\infty}\int_A\phi_n(x,y)\phi_p(x,y)\theta_n(t)dA \quad (11)$$

Considering the orthogonality of $\phi_n(x,y)$, we have

$$\theta_n(t)=\frac{1}{P^*}\sum_{n=1}^{\infty}\int_A \Big\{R_0\big[\phi_{n,xx}(x,y)\phi_p(x,y)U_{n,tt}(t)+\phi_{n,yy}(x,y)\phi_p(x,y)U_{n,tt}(t)\big]+\frac{N_x}{\mu}\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)$$
$$+\frac{N_y}{\mu}\phi_{n,yy}(x,y)\phi_p(x,y)U_n(t)-\frac{F_0}{\mu}\phi_n(x,y)\phi_p(x,y)U_n(t)+\frac{G_0}{\mu}\big[\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)+\phi_{n,yy}(x,y)\phi_p(x,y$$
$$+\sum_{i=1}^{N}\Big[\frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_it)\delta(y-s)-\frac{M_i}{\mu}\Big(\phi_n(x,y)\phi_p(x,y)U_{n,tt}(t)+2c_i\phi_{n,x}(x,y)\phi_p(x,y)U_{n,t}(t)$$
$$+c_i^2\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)\Big)\delta(x-c_it)\delta(y-s)\Big]\Big\}dA \quad (12)$$

where $P^*=\int_A\phi_p^2 dA$

Using (12), equation (7), taking into account (4) and (5), can be written as

$$\phi_n(x,y)\left[\frac{D\omega_n^4}{\mu}U_n(t)+U_{n,tt}(t)\right]=\frac{\phi_n(x,y)}{P^*}\sum_{q=1}^{\infty}\int_A \Big\{R_0\big[\phi_{q,xx}(x,y)\phi_p(x,y)U_{q,tt}(t)+\phi_{q,yy}(x,y)\phi_p(x,y)U_{q,tt}(t)\big]$$
$$+\frac{N_x}{\mu}\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)+\frac{N_y}{\mu}\phi_{q,yy}(x,y)\phi_p(x,y)U_q(t)-\frac{F_0}{\mu}\phi_q(x,y)\phi_p(x,y)U_q(t)$$
$$+\frac{G_0}{\mu}\big[\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)+\phi_{q,yy}(x,y)\phi_p(x,y)U_q(t)\big]+\sum_{i=1}^{N}\Big[\frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_it)\delta(y-s)$$
$$-\frac{M_i}{\mu}\Big(\phi_q(x,y)\phi_p(x,y)U_{q,tt}(t)+2c_i\phi_{q,x}(x,y)\phi_p(x,y)U_{q,t}(t)+c_i^2\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)\Big)\delta(x-c_it)\delta(y-s) \quad (13)$$

Equation (13) must be satisfied for arbitrary x, y and this is possible only when

$$U_{n,tt}(t)+\frac{D\omega_n^4}{\mu}U_n(t)=\frac{1}{P^*}\sum_{q=1}^{\infty}\int_A \Big\{R_0\big[\phi_{q,xx}(x,y)\phi_p(x,y)U_{q,tt}(t)+\phi_{q,yy}(x,y)\phi_p(x,y)U_{q,tt}(t)\big]$$
$$+\frac{N_x}{\mu}\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)+\frac{N_y}{\mu}\phi_{q,yy}(x,y)\phi_p(x,y)U_q(t)-\frac{F_0}{\mu}\phi_q(x,y)\phi_p(x,y)U_q(t)$$
$$+\frac{G_0}{\mu}\big[\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)+\phi_{q,yy}(x,y)\phi_p(x,y)U_q(t)\big]+\sum_{i=1}^{N}\Big[\frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_it)\delta(y-s)$$
$$-\frac{M_i}{\mu}\Big(\phi_q(x,y)\phi_p(x,y)U_{q,tt}(t)+2c_i\phi_{q,x}(x,y)\phi_p(x,y)U_{q,t}(t)+c_i^2\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)\Big)\delta(x-c_it)\delta(y-s) \quad (14)$$

The system in equation (14) is a set of coupled ordinary differential equations.

The Dirac-Delta function is expressed in the Fourier cosine series as (Oni and Awodola, 2011)

$$\delta(x-c_i t)=\frac{1}{L_X}\left[1+2\sum_{j=1}^{\infty}\cos\frac{j\pi c_i t}{L_X}\cos\frac{j\pi x}{L_X}\right] \text{ and } \delta(y-s)=\frac{1}{L_Y}\left[1+2\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}\cos\frac{k\pi y}{L_Y}\right]$$

(15)

We shall consider only one mass M traveling with uniform velocity c along the line y = s. Thus for the single mass M equation (14) reduces to

$$\frac{d^2U_n(t)}{dt^2}+\alpha_n^2U_n(t)-\frac{1}{P^*}\sum_{q=1}^{\infty}\left\{R_0P_1^*\frac{d^2U_q(t)}{dt^2}-\frac{1}{\mu}\left(F_0P_2^*-N_xP_{1A}^*-N_yP_{1B}^*-G_0P_1^*\right)U_q(t)\right.$$
$$-\Gamma\left[2\left(\frac{P_3^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\frac{d^2U_q(t)}{dt^2}\right.$$
$$+4c\left(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dU_q(t)}{dt}$$
$$\left.\left.+2c^2\left(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)U_q(t)\right]\right\}$$
$$=\frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (16)$$

where $\Gamma=\dfrac{M}{L_XL_Y\mu}$, $\alpha_n^2=\dfrac{D\omega_n^4}{\mu}$,

$$P_{1A}^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,\quad P_{1B}^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,yy}(x,y)\phi_p(x,y)\,dy\,dx$$

$$P_1^*=\int_0^{L_X}\int_0^{L_Y}\left[\phi_{n,xx}(x,y)+\phi_{n,yy}(x,y)\right]\phi_p(x,y)\,dy\,dx,\qquad P_2^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,\quad P_3^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,\quad P_3^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,\quad P_4^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,\quad P_4^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,\quad P_5^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx \quad and \quad P_5^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,d$$

The second order coupled differential equation (16) is the transformed equation governing the problem of a prestressed rectangular plate on a Pasternak elastic foundation. We shall then solve the equation (16) when the plate has simple supports at all its edges. $\phi_n(x,y)$ are assumed to be the products of the functions $\psi_{ni}(x)$ and $\psi_{nj}(y)$ which are the beam functions in the directions of x and y axes respectively (Lee and Ng 1996, Oni and Awodola 2011) . That is

$$\phi_n(x,y) = \psi_{ni}(x)\psi_{nj}(y) \tag{17}$$

These beam functions can be defined respectively, as

$$\psi_{ni}(x) = \sin\frac{\Omega_{ni}x}{L_X} + A_{ni}\cos\frac{\Omega_{ni}x}{L_X} + B_{ni}\sinh\frac{\Omega_{ni}x}{L_X} + C_{ni}\cosh\frac{\Omega_{ni}x}{L_X} \tag{18}$$

and

$$\psi_{nj}(x) = \sin\frac{\Omega_{nj}y}{L_Y} + A_{nj}\cos\frac{\Omega_{nj}y}{L_Y} + B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y} + C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y} \tag{19}$$

where A_{ni}, A_{nj}, B_{ni}, B_{nj}, C_{ni} and C_{nj} are constants determined by the boundary conditions. Ω_{ni} and Ω_{nj} are called the mode frequencies.

When use is made of the boundary conditions (3b – 3i) on (18) and (19), it is easy to show that

$A_{ni} = 0$, $B_{ni} = 0$, $C_{ni} = 0$, and $\Omega_{ni} = n_i\pi$ (20a)

$A_{nj} = 0$, $B_{nj} = 0$, $C_{nj} = 0$ and $\Omega_{nj} = n_j\pi$ (20b)

Similarly,

$A_{pi} = 0$, $B_{pi} = 0$, $C_{pi} = 0$, apd $\Omega_{pi} = p_i\pi$ (20c)

$A_{pj} = 0$, $B_{pj} = 0$, $C_{pj} = 0$ apd $\Omega_{pj} = p_j\pi$ (20d)

Substituting equations (20a), (20b), (20c) and (20d) into the transformed equation (16) to obtain the transformed equation for a prestressed rectangular plate resting on a Pasternak elastic foundation and having simple supports at all i ts edges, we have

$$\frac{d^2U_n(t)}{dt^2} + \alpha_n^2 U_n(t) - \frac{1}{P^*}\sum_{q=1}^{\infty}\left\{\frac{-R_0L_XL_Y\pi^2}{4}\left(\frac{q^2}{L_X^2}+\frac{q^2}{L_Y^2}\right)\frac{d^2U_q(t)}{dt^2} - \frac{1}{\mu}\left(F_0\frac{L_y}{2} - N_xP_{1A}^* - N_yP_{1B}^* - G_0P_1^*\right)\right.$$
$$\left. -\Gamma\left[\frac{L_XL_Y}{4}\frac{d^2U_q(t)}{dt^2} + 2cL_Y\left(\frac{qp_i}{p_i^2-q^2} + \sum_{j=1}^{\infty}\frac{q\pi}{L_X}\tau(j)\cos\frac{j\pi ct}{L_X}\right)\frac{dU_q(t)}{dt} - \frac{(cq\pi)^2L_Y}{4L_X}U_q(t)\right]\right\}$$
$$= \frac{Mg}{P^*\mu}\sin\frac{p_j\pi s}{L_Y}\sin\frac{p_i\pi ct}{L_X} \tag{21}$$

where $\tau(j) = \dfrac{8p_i[p_i^2 - j^2 - q^2]}{j^4 + q^4 + p_i^4 - 2[j^2p_i^2 + j^2q^2 + p_i^2q^2]}$

Equation (21) is now the fundamental equation of our problem when the prestressed rectangular plate resting on Pasternak foundation has simple support at all its edges. In what follows, we shall discuss two cases of the equation.

CASE I: SIMPLY SUPPORTED PRESTRESSED PLATE TRAVERSED BY MOVING FORCE :

When the inertia effect of the moving mass M is neglected, that is, when $\Gamma = 0$ in equation (21), we have the moving force problem associated with the system. Thus the differential equation is given by

$$\frac{d^2U_n(t)}{dt^2} + \alpha_n^2 U_n(t) - \frac{1}{P^*}\sum_{q=1}^{\infty}\left\{\frac{-R_0L_XL_Y\pi^2}{4}\left(\frac{q^2}{L_X^2}+\frac{q^2}{L_Y^2}\right)\frac{d^2U_q(t)}{dt^2}\right.$$
$$\left. -\frac{1}{\mu}\left(F_0\frac{L_y}{2} - N_xP_{1A}^* - N_yP_{1B}^* - G_0P_1^*\right)U_q(t)\right\} = \frac{Mg}{P^*\mu}\sin\frac{p_j\pi s}{L_Y}\sin\frac{p_i\pi ct}{L_X} \tag{22}$$

Using the Struble's asymptotic technique (Oni and Awodola 2011), the moving force problem (22) for the simply supported prestressed rectangular plate is reduced to the non-homogeneous ordinary differential equation given as

$$\frac{d^2U_n(t)}{dt^2} + \gamma_{mf}^2 U_n(t) = K_0 \sin\frac{p_j \pi s}{L_Y} \sin\frac{p_i \pi ct}{L_X} \tag{23}$$

where

$$\gamma_{mf} = \gamma_m \left[1 - \frac{\varepsilon_0 L_X L_Y \pi^2}{8}\left(\frac{n_i^2}{L_X^2} + \frac{n_j^2}{L_Y^2}\right)\right] \tag{24}$$

is the modified frequency of the system.

$$\gamma_m = \alpha_n + \frac{\lambda}{2\alpha_n},\ \lambda = \frac{1}{P^*\mu}\left(F_0\frac{L_y}{2} - N_x P_{1A}^* - N_y P_{1B}^* - G_0 P_1^*\right),\ \varepsilon_0 = \frac{R_0}{P^*} \text{ and } K_0 = \frac{Mg}{P^*\mu}$$

When equation (23) is solved in conjunction with the initial conditions, one obtains expression for $U_n(t)$. Thus in view of equation (4), one obtains

$$H(x,y,t) = \sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty} \frac{K_0 \sin\frac{p_j\pi s}{L_Y}}{\gamma_{mf}[\gamma_{mf}^2 - (p_i\pi c/L_X)^2]}\left[\gamma_{mf}\sin\frac{p_i\pi ct}{L_X} - \frac{p_i\pi c}{L_X}\sin\gamma_{mf}t\right]\sin\frac{n_i\pi x}{L_X}\sin\frac{n_j\pi y}{L_Y} \tag{25}$$

Equation (25) is the transverse -displacement response to a moving force of a simply supported and prestressed rectangular plate on a bi-parametric (Pasternak) elastic foundation.

CASE II: SIMPLY SUPPORTED PRESTRESSED PLATE TRAVERSED BY MOVING MASS:

In this section we seek the solution to the entire equation (21) when no term of the equation is neglected. To solve this problem, we use the modified asymptotic method of Struble already alluded to (Oni and Awodola 2011). To this end, we rearrange equation (21) to take the form

$$\frac{d^2U_n(t)}{dt^2} - \frac{2cL_Y\eta_0\left(\frac{n_i p_i}{p_i^2 - n_i^2} + \sum_{j=1}^{\infty}\frac{n_i\pi}{L_X}\tau(j)\cos\frac{j\pi ct}{L_X}\right)}{1-\eta_0\left(\frac{L_X L_Y}{4}\right)}\frac{dU_n(t)}{dt} + \frac{\gamma_{mf}^2 + \frac{\eta_0 (cn_i\pi)^2 L_Y}{4L_X}}{1-\eta_0\left(\frac{L_X L_Y}{4}\right)}U_n(t)$$

$$-\frac{\eta_0}{1-\eta_0\left(\frac{L_X L_Y}{4}\right)}\sum_{\substack{q=1\\q\neq n}}^{\infty}\left[\frac{L_X L_Y}{4}\frac{d^2T_q(t)}{dt^2} + 2cL_Y\left(\frac{qp_i}{p_i^2 - q^2} + \sum_{j=1}^{\infty}\frac{q\pi}{L_X}\tau(j)\cos\frac{j\pi ct}{L_X}\right)\frac{dU_q(t)}{dt} - \frac{(cq\pi)^2 L_Y}{4L_X}U_q(t\right.$$

$$= \frac{\eta_0 g L_X L_Y}{P^*\left[1-\eta_0\left(\frac{L_X L_Y}{4}\right)\right]}\sin\frac{p_j\pi s}{L_Y}\sin\frac{p_i\pi ct}{L_X} \tag{26}$$

where Γ has been written as a function of the mass ratio η_0.

Thus, Using the Struble's asymptotic technique, the modified frequency corresponding to the frequency of the free system due to the presence of the moving mass is

$$\beta_f = \gamma_{mf}\left[1 - \frac{\eta_0}{2}\left(1 + \frac{(cn_i\pi)^2}{\gamma_{mf}^2 L_X^2}\right)\right] \tag{27}$$

retaining terms to $o(\eta_0)$ only.

Therefore, the differential operator which acts on $U_n(t)$ and $U_q(t)$ is replaced by the equivalent free system operator defined by the modified frequency β_f. That is, equation (26) becomes

$$\frac{d^2U_n(t)}{dt^2} + \beta_f^2 U_n(t) = G_0 \sin\frac{n_j \pi s}{L_Y} \sin\frac{n_i \pi ct}{L_X} \quad (28)$$

where $G_0 = \frac{\eta_0 g L_X L_Y}{P^*}$ (29)

Hence when equation (28) is solved in conjunction with the initial conditions, one obtains expression for $U_n(t)$. Thus in view of equation (4), we have

$$H(x,y,t) = \sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty} \frac{G_0 \sin\frac{p_j \pi s}{L_Y}}{\beta_f [\beta_f^2 - (p_i \pi c / L_X)^2]} \left[\beta_f \sin\frac{p_i \pi ct}{L_X} - \frac{p_i \pi c}{L_X} \sin \beta_f t\right] \sin\frac{n_i \pi x}{L_X} \sin\frac{n_j \pi y}{L_Y} \quad (30)$$

Equation (30) is the transverse-displacement response to a moving mass of a simply supported prestressed rectangular plate on a Pasternak elastic foundation.

4. DISCUSSION OF THE ANALYTICAL SOLUTIONS

It is desirable to examine the phenomenon of resonance. Thus, equation (25) clearly shows that the simply supported prestressed rectangular plate on a Pasternak elastic foundation and traversed by a moving force reaches a state of resonance whenever

$$\gamma_{mf} = \frac{p_i \pi c}{L_X} \quad (31)$$

while equation (30) shows that the same plate under the action of a moving mass experiences resonance when

$$\beta_f = \frac{p_i \pi c}{L_X} \quad (32)$$

where
$$\beta_f = \gamma_{mf}\left[1 - \frac{\eta_0}{2}\left(1 + \frac{(cn_i\pi)^2}{\gamma_{mf}^2 L_X^2}\right)\right] \quad (33)$$

Equations (32) and (33) imply that $\gamma_{mf}\left[1 - \frac{\eta_0}{2}\left(1 + \frac{(cn_i\pi)^2}{G_f^2 L_X^2}\right)\right] = \frac{p_i \pi c}{L_X}$ (34)

Since $\left[1 - \frac{\eta_0}{2}\left(1 + \frac{(cn_i\pi)^2}{G_f^2 L_X^2}\right)\right] < 1$ for all n_i, it can be deduced from equation (34) that, for the same natural frequency, the critical speed (and the natural frequency) for the system of a simply supported and prestressed rectangular plate resting on a Pasternak elastic foundation and traversed by a moving mass is smaller than that of the same system traversed by a moving force. Thus, for the same natural frequency of the plate, resonance is reached earlier when we consider the moving mass system than when we consider the moving force system.

5. NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

For the simply supported prestressed plate resting on Pasternak elastic foundation, a rectangular plate of length L_Y = 0.914m and breadth L_X = 0.457m is considered. It is assumed that the mass travels at the constant velocity 0.8123m/s. Furthermore, values for E, S and Γ are chosen to be 2.109×10^9kg/m^2, 0.4m and 0.2 respectively. For various values of the axial forces (N_x and N_y), rotatory inertia R_0, shear modulus G_0, and the mass ratio Γ, the deflections of the simply supported prestressed plate are calculated and plotted against time t.

Figures 1 and 2 display the effects of axial forces N_x and N_y respectively on the transverse deflection of the simply supported prestressed rectangular plate for the case of moving mass. The graphs show that the response amplitudes decrease as both N_x and N_y increase.

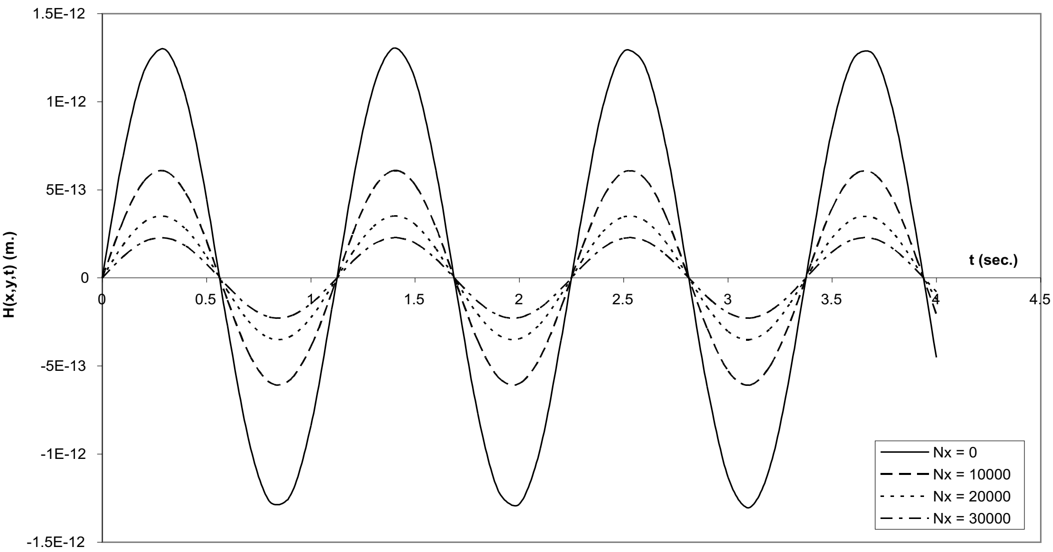

Fig.1: Deflection profile of prestressed simply supported plate on bi-parametric elastic foundation and traversed by moving mass for various values of axial force Nx.

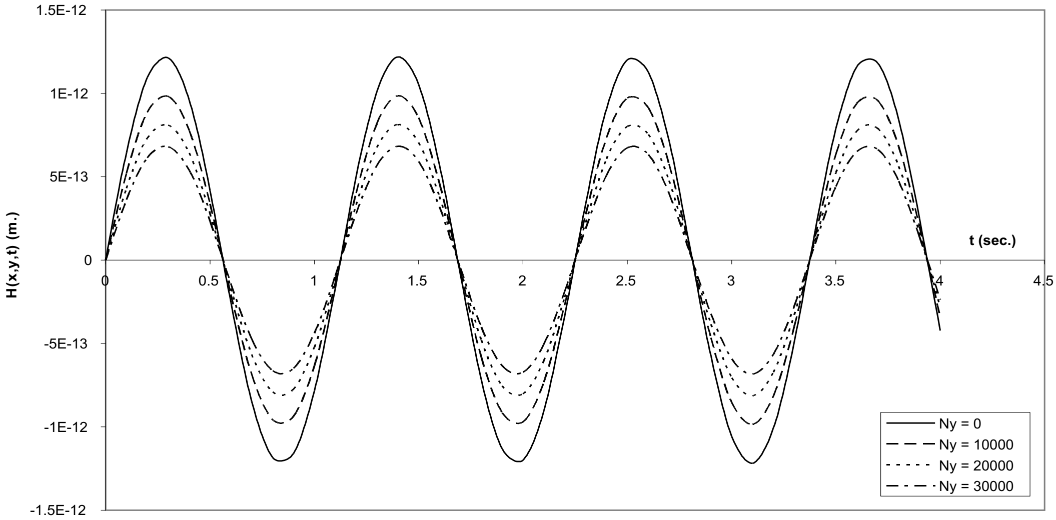

Fig.2: Deflection profile of prestressed simply supported plate on bi-parametric elastic foundation and traversed by moving mass for various values of axial force Ny.

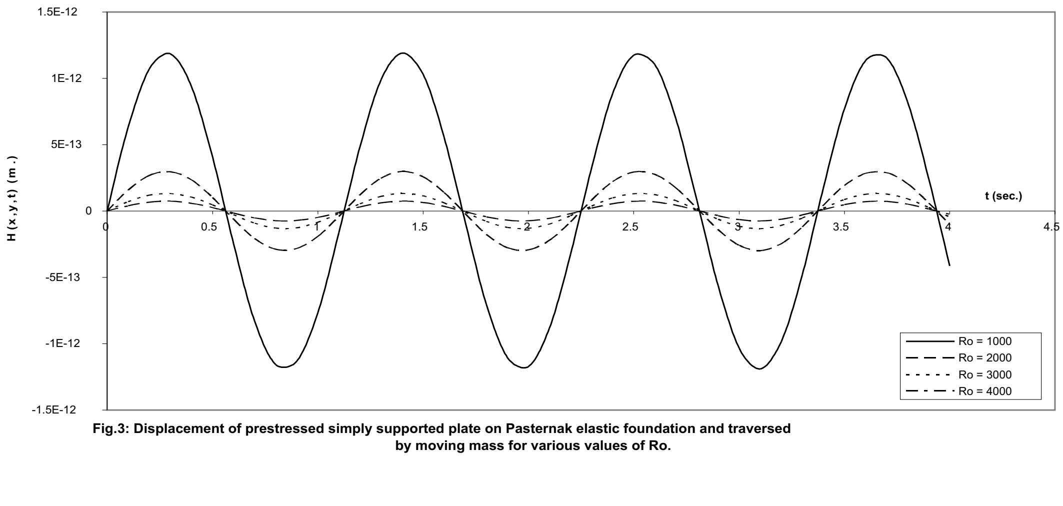

Fig.3: Displacement of prestressed simply supported plate on Pasternak elastic foundation and traversed by moving mass for various values of Ro.

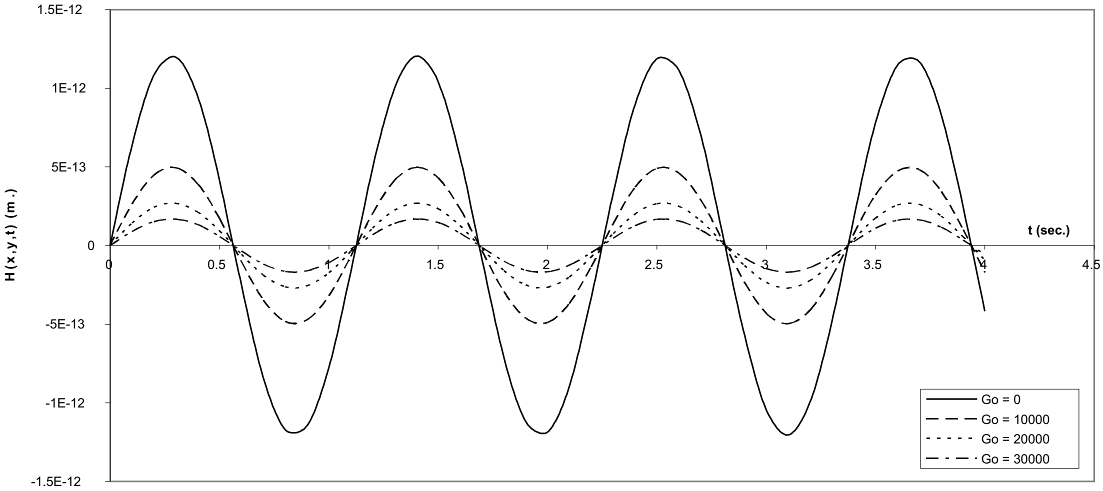

Fig.4: Displacement of prestressed simply supported plate on Pasternak elastic foundation and traversed by moving mass for various values of Go.

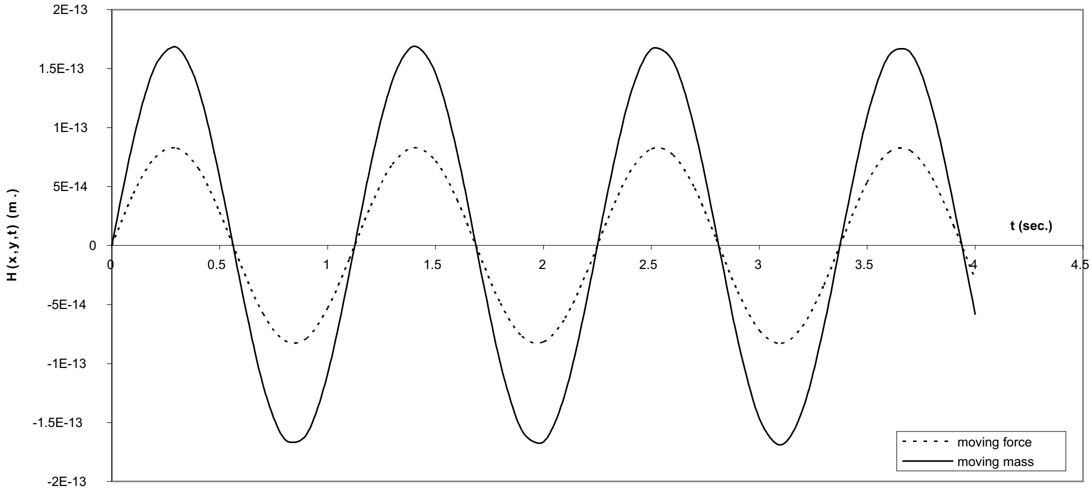

Fig.5: Comparison of moving force and moving mass cases of prestressed simply supported rectangular plate resting on Pasternak elastic foundation.

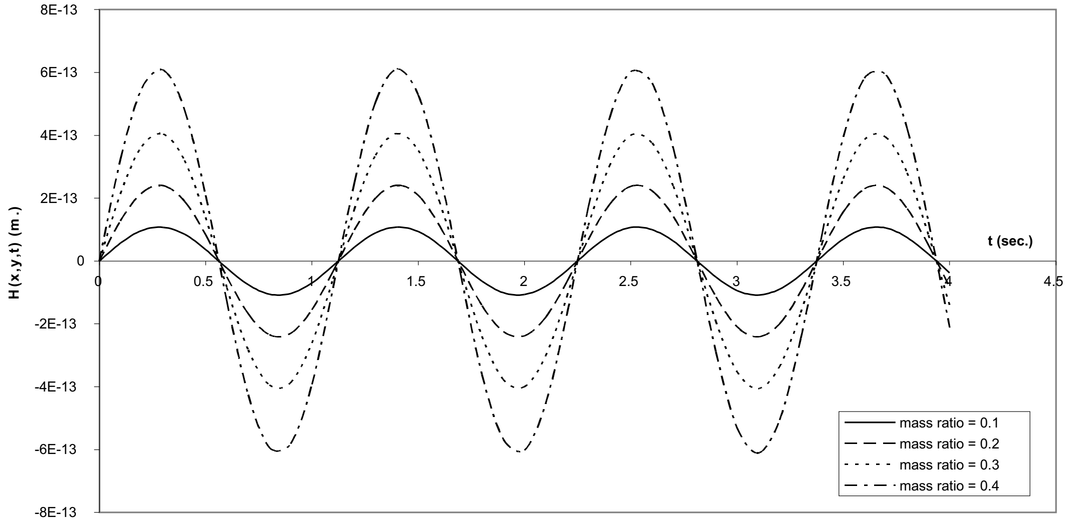

Fig.6: Displacement profile of moving mass of prestressed simply supported rectangular plate resting on Pasternak elastic foundation for various values of mass ratio.

Figures 3 and 4 show the effects of rotatory inertia R_0 and shear modulus G_0 respectively on the transverse deflection of the prestressed plate for the case of moving mass. The graphs show that the response amplitudes decrease as both R_0 and G_0 increase.

Figure 5 compares the displacement curves of the moving force and moving mass for a simply supported prestressed rectangular plate. Clearly, the response amplitude of a moving mass is greater than that of a moving force problem.

Figure 6 displays the effect of mass ratio on the transverse deflection of the simply supported prestressed rectangular plate. The graph shows that the response amplitude increases as the value of increases. This implies that the moving mass solution approaches the moving force solution as the mass ratio approaches zero.

6. Conclusion

The objective of this work has been to study the problem of the dynamic response to moving concentrated masses of prestressed and simply supported rectangular plates on Pasternak elastic foundations. In particular, the closed form solutions of the fourth order partial differential equations with variable and singular coefficients of the rectangular plate is obtained for both cases of moving force and moving mass. The solution technique is based on the technique of Shadnam et al (2001) which was used to remove the singularity in the governing fourth order partial differential equation and to reduce it to a sequence of coupled second order differential equations. These coupled second order differential equations were then simplified using the modified Struble's asymptotic technique. The methods of integral transformation and the convolution theory were then employed to obtain the analytical solutions of the two-dimensional dynamical problem.

These solutions were analyzed and resonance conditions were obtained for the problem. The analyses carried out show that, for the same natural frequency, the critical speed (and the natural frequency) for the system of the prestressed and simply supported rectangular plate resting on Pasternak elastic foundation and traversed by a moving mass is smaller than that of the same system traversed by a moving force. Thus, for the same natural frequency of the plate, the resonance is reached earlier when we consider the moving mass system than when we consider the moving force system. The analyses show that the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

The results in plotted curves show that as each of the axial forces N_x and N_y increases, the response amplitudes of the plates decrease for both cases of moving force and moving mass problems for the prestressed simply supported plate. Also, the response amplitudes of the plate decrease as the value of the shear modulus G_o increases, the same effect is obtained as the rotatory inertia correction factor R_o increases. Furthermore, the response amplitude for the moving mass problem is greater than that of the moving force problem for fixed values of N_x and N_y, implying that resonance is reached earlier in moving mass problem than in moving force problem, making it unsafe to rely on the moving force solutions. Also, as the mass ratio Γ approaches zero, the response amplitudes of the moving mass problem approach that of the moving force problem of the prestressed and simply supported rectangular plate resting on uniform Pasternak elastic foundation.

Finally, the results in this work agree with what obtain in literature (Oni 1991, Gbadeyan and Oni 1995, Oni and Awodola 2005, Oni and Ogunbamike 2011, Awodola and Omolofe 2016). Hence the method employed in this work is accurate and the solutions are convergent.

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