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# Dynamic Behaviour Under Moving Masses of Prestressed and Elastically Supported Plates Resting on Winkler Foundation

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Prestress,
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Moving Force,
Moving Mass,
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Critical speed,
Mass Ratio.

**A B S T R A C T**

*The dynamic behaviour of prestressed and elastically supported rectangular plates under moving concentrated masses and resting on Winkler elastic foundation is investigated in this work. This problem involves non-classical boundary conditions; it is solved using a technique based on separation of variables and a modification of Struble's technique, the solution is illustrated with two common examples of non-classical boundary conditions often encountered in engineering practice. The numerical results in plotted curves show that the response amplitudes of the plates decrease as the value of the axial force in x-direction (Nx) increases, the response amplitudes also decrease as axial force in y-direction (Ny) increase for both cases of moving force and moving mass problems of the prestressed and elastically supported rectangular plate resting on Winkler elastic foundation for the illustrative examples considered. The deflection of the plate also decreases as the value of the rotatory inertia correction factor R0 increases. Also, for fixed values of Nx and Ny, the transverse deflections of the rectangular plates under the actions of moving masses are higher than those when only the force effects of the moving loads are considered and the critical speed for the moving mass problem is reached prior to that of the moving force problem. It is further shown that the moving force solution is not a safe approximation to the moving mass problem which implies that it is risky to rely on a design based on the moving force solution. The response amplitudes of the moving mass problem increase with increasing mass ratio and approach the response amplitudes of the moving force as the mass ratio approaches zero for the prestressed and elastically supported rectangular plates resting on Winkler elastic foundation.*

## 1. Introduction

The effects of moving loads on solid bodies are dual [1]. On one hand is the gravitational effect of the moving load while on the other hand is the inertia effect of the mass of the load on the vibrating solid bodies. When the inertia effect of the moving load is considered, the governing differential equation of motion becomes complex and cumbersome and no longer has constant coefficients. In particular, the coefficients become variable and singular. If the inertia effect of the moving load is neglected, the problem is termed moving force problem and when it is retained, it is termed moving mass problem.

Many researchers have made tremendous efforts in analyzing the dynamic response of elastic structures under the action of moving masses [2-6]. In most analytical studies in Engineering and Mathematical Physics, structural members are commonly modeled as a beam or as a plate. The problem of determining the dynamic response of structures (beams or plates) under the action of moving concentrated masses has been almost exclusively reserved for structures having the normal ideal boundary conditions called the classical boundary conditions. Such ideal boundary conditions include among others, Clamped edge, Free edge, Simply supported edge and Sliding edge boundary conditions. For practical applications in many cases, it is more realistic to consider non-classical boundary conditions because the ideal boundary conditions can seldom be realized. A common example is the elastically supported end conditions. As a problem of this kind, Wilson [7] studied the response of a cantilever plate strip restrained elastically against rotation and subjected to a moving normal line load. In a later development, Saito et al [8] presented a theoretical analysis of the steady state response of a plate strip constrained elastically along its edges against rotation and translation under the action of a moving transverse line load. The first five speeds of the applied load for which a resonance effect occurs in the system are plotted as functions of the edge constraint parameters.

In the recent time, several researchers have made efforts in the study of dynamics of structures under moving loads [9 - 16]. Oni and Awodola [17] considered the dynamic behaviour under moving concentrated masses of elastically supported finite Bernoulli-Euler beam on Winkler foundation. The technique was based on the generalized finite integral transform method.

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Engineers often create artificial stresses in structures before loading, such artificial stresses are forces which may act axially or otherwise. When they act axially, they are called axial forces. The artificial stresses are also called prestress. The aim of prestressed structures is to limit tensile stresses and hence flexural cracking or bending in the structure under working conditions. If the structure is subjected to a force parallel to its axes in addition to the lateral loading, the local equilibrium of forces is altered because the axial force interacts with the lateral displacement to produce an additional term, Clough and Penziens [18]. This additional term due to the axial force increases the complexity of the problem.

In all the aforementioned investigations, considerations have been limited to cases of one-dimensional (beam) problems. Where two-dimensional (plate) problems have been considered, no considerations have been given to the class of dynamical problems in which the plate is prestressed, especially the influence of the prestress (axial force) on such dynamical system with non-classical boundary conditions. Therefore, this study investigates the influence of axial force on the response to moving concentrated masses of prestressed and elastically supported rectangular plates resting on Winkler elastic foundation.

## 2. Governing Equation

The equation governing the dynamic transverse displacement Z(x,y,t) of a prestressed rectangular plate when it is resting on a constant Winkler elastic foundation and traversed by concentrated mass M moving with velocity c (issuing from point y = s on the y – axis) is the fourth order partial differential equation given by:

$$\frac{Eh^2\nabla^4 Z(x,y,t)}{12(1-\upsilon)}+\mu\frac{\partial^2 Z(x,y,t)}{\partial t^2}=\mu R_0\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]Z(x,y,t)$$
$$+\left[N_x\frac{\partial^2 Z(x,y,t)}{\partial x^2}+N_y\frac{\partial^2 Z(x,y,t)}{\partial y^2}\right]-F_0 Z(x,y,t)+\left[Mg\delta(x-ct)\delta(y-s)\right.$$
$$\left.-M\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial t\partial x}+c^2\frac{\partial^2}{\partial x^2}\right)Z(x,y,t)\delta(x-ct)\delta(y-s)\right] \quad (1)$$

where $\nabla^2$ is the two-dimensional Laplacian operator, h is the plate's thickness, E is the Young's Modulus, $\nu$ is the Poisson's ratio ($\nu<1$), $\mu$ is the mass per unit area of the plate, $N_x$ and $N_y$ are the axial forces in x and y directions respectively, $R_0$ is the Rotatory inertia correction factor, $F_0$ is the foundation's stiffness, x and y are respectively the spatial coordinates in x and y directions and t is the time coordinate.

## 3. Analytical Approximate Solution

In the first instance, we consider rectangular plate elastically supported at edges $y = 0, y = L_Y$ with simple support at edges $x = 0, x = L_X$.

If $\frac{\partial^2 Z(x,y,t)}{\partial x^2}\equiv Z_{xx}(x,y,t)$, $\frac{\partial^2\Psi_{ni}(x,t)}{\partial x^2}\equiv\Psi_{ni,xx}(x,t)$

and so on, the boundary conditions can be written as [19]

$$Z_{xx}(0,y,t)-k_1Z_x(0,y,t)=0,\quad Z_{xx}(L_X,y,t)-k_1Z_x(L_X,y,t)=0 \quad (10)$$

$$Z_{yy}(x,0,t)-k_1Z_y(x,0,t)=0,\quad Z_{yy}(x,L_Y,t)-k_1Z_y(x,L_Y,t)=0 \quad (11)$$

$$Z_{xxx}(0,y,t)+k_2Z(0,y,t)=0,\quad Z_{xxx}(L_X,y,t)+k_2Z(L_X,y,t)=0 \quad (12)$$

$$Z_{yyy}(x,0,t)+k_2Z(x,0,t)=0,\quad Z_{yyy}(x,L_Y,t)+k_2Z(x,L_Y,t)=0 \quad (13)$$

and for normal modes

$$\Psi_{ni,xx}(0)-k_1\Psi_{ni,x}(0)=0,\quad \Psi_{ni,xx}(L_Y)-k_1\Psi_{ni,x}(L_Y)=0 \quad (14)$$

$$\Psi_{nj,yy}(0)-k_1\Psi_{nj,y}(0)=0,\quad \Psi_{nj,yy}(L_Y)-k_1\Psi_{nj,y}(L_Y)=0 \quad (15)$$

$$\Psi_{ni,xxx}(0)+k_2\Psi_{ni}(0)=0,\quad \Psi_{ni,xxx}(L_Y)+k_2\Psi_{ni}(L_Y)=0 \quad (16)$$

$$\Psi_{nj,yyy}(0)+k_2\Psi_{nj}(0)=0,\quad \Psi_{nj,yyy}(L_Y)+k_2\Psi_{nj}(L_Y)=0 \quad (17)$$

where $k_1$ and $k_2$ are the stiffness against rotation and the stiffness against translation respectively. The initial conditions, without any loss of generality, is taken as

$$Z(x,y,0)=0=\frac{\partial Z(x,y,0)}{\partial t} \quad (18)$$

Evidently, an exact closed form solution of the above fourth order partial differential equation (1) does not exist. Consequently, an approximate solution is sought. Thus, the technique based on separation of variable described in [9] is employed. This versatile technique requires that the solution of equation (1) takes the form

$$Z(x,y,t)=\sum_{n=1}^{\infty}\phi_n(x,y)U_n(t) \quad (19)$$

where $\phi_n$ are the known eigen functions of the plate with the same boundary conditions and have the form of [20]

$$\nabla^4\phi_n-\omega_n^4\phi_n=0 \quad (20)$$

where $\omega_n^4=\frac{\Omega_n^2\mu}{D}$ (21)

$\Omega_n$, n = 1, 2, 3, … , are the natural frequencies of the dynamical system and $U_n(t)$ are amplitude functions which have to be calculated.

In order to solve the equation (1), it is rewritten as

$$\frac{D}{\mu}\nabla^4 Z(x,y,t)+\frac{\partial^2 Z(x,y,t)}{\partial t^2}=R_0\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]Z(x,y,t)$$
$$+\left[N_x\frac{\partial^2 Z(x,y,t)}{\partial x^2}+N_y\frac{\partial^2 Z(x,y,t)}{\partial y^2}\right]-\frac{F_0}{\mu}Z(x,y,t)+\left[\frac{Mg}{\mu}\delta(x-ct)\delta(y-s)\right.$$
$$\left.-\frac{M}{\mu}\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial t\partial x}+c^2\frac{\partial^2}{\partial x^2}\right)Z(x,y,t)\delta(x-ct)\delta(y-s)\right] \quad (22)$$

where $D=\frac{Eh^2}{12(1-\nu)}$ is the bending rigidity of the plate.

Rewriting the right hand side of equation (22) in the form of a series, we have

$$R_0\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]Z(x,y,t)+\left[N_x\frac{\partial^2 Z(x,y,t)}{\partial x^2}+N_y\frac{\partial^2 Z(x,y,t)}{\partial y^2}\right]$$
$$-\frac{F_0}{\mu}Z(x,y,t)+\left[\frac{Mg}{\mu}\delta(x-ct)\delta(y-s)\right.$$
$$\left.-\frac{M}{\mu}\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial t\partial x}+c^2\frac{\partial^2}{\partial x^2}\right)Z(x,y,t)\delta(x-ct)\delta(y-s)\right]=\sum_{n=1}^{\infty}\phi_n(x,y)B_n(t)$$

When equation (19) is used in equation (23) we have

$$\sum_{n=1}^{\infty}\left\{R_0\left[\phi_{n,xx}(x,y)U_{n,tt}(t)+\phi_{n,yy}(x,y)U_{n,tt}(t)\right]-\frac{F_0}{\mu}\phi_n(x,y)U_n(t)\right.$$
$$+\frac{N_x}{\mu}\phi_{n,xx}(x,y)U_n(t)+\frac{N_y}{\mu}\phi_{n,yy}(x,y)U_n(t)+\frac{Mg}{\mu}\delta(x-ct)\delta(y-s)$$

$$-\frac{M}{\mu}\Big(\phi_n(x,y)U_{n,tt}(t)+2c\phi_{n,x}(x,y)U_{n,t}(t)$$

$$+c^2\phi_{n,xx}(x,y)U_n(t)\Big)\delta(x-ct)\delta(y-s)\Big]\Big\}=\sum_{n=1}^{\infty}\phi_n(x,y)B_n(t) \qquad (24)$$

Multiplying both sides of equation (24) by $\phi_p(x,y)$ and integrating on area A of the plate, we have

$$\sum_{n=1}^{\infty}\int_A \Big\{R_0\Big[\phi_{n,xx}(x,y)\phi_p(x,y)U_{n,tt}(t)+\phi_{n,yy}(x,y)\phi_p(x,y)U_{n,tt}(t)\Big]$$

$$+\frac{N_x}{\mu}\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)+\frac{N_y}{\mu}\phi_{n,yy}(x,y)\phi_p(x,y)U_n(t)$$

$$-\frac{F_0}{\mu}\phi_n(x,y)\phi_p(x,y)U_n(t)+\frac{Mg}{\mu}\phi_p(x,y)\delta(x-ct)\delta(y-s)$$

$$-\frac{M}{\mu}\Big(\phi_n(x,y)\phi_p(x,y)U_{n,tt}(t)+2c\phi_{n,x}(x,y)\phi_p(x,y)U_{n,t}(t)$$

$$+c^2\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)\Big)\delta(x-ct)\delta(y-s)\Big]\Big\}dA$$

$$=\sum_{n=1}^{\infty}\int_A \phi_n(x,y)\phi_p(x,y)B_n(t)\,dA \qquad ...........(25)$$

Considering the orthogonality of $\phi_n(x,y)$, we have

$$B_n(t)=\frac{1}{P^*}\sum_{n=1}^{\infty}\int_A \Big\{R_0\Big[\phi_{n,xx}(x,y)\phi_p(x,y)U_{n,tt}(t)+\phi_{n,yy}(x,y)\phi_p(x,y)U_{n,tt}(t)\Big]$$

$$+\frac{N_x}{\mu}\phi_{n,xx}(x,y)U_n(t)\phi_p(x,y)+\frac{N_y}{\mu}\phi_{n,yy}(x,y)\phi_p(x,y)U_n(t)$$

$$-\frac{F_0}{\mu}\phi_n(x,y)\phi_p(x,y)U_n(t)+\frac{Mg}{\mu}\phi_p(x,y)\delta(x-ct)\delta(y-s)$$

$$-\frac{M}{\mu}\Big(\phi_n(x,y)\phi_p(x,y)U_{n,tt}(t)+2c\phi_{n,x}(x,y)\phi_p(x,y)U_{n,t}(t)$$

$$+c^2\phi_{n,xx}(x,y)\phi_p(x,y)U_n(t)\Big)\delta(x-ct)\delta(y-s)\Big]\Big\}dA \qquad (26)$$

where $P^*=\int_A \phi_p^2\,dA$

Using (26) and taking into account (19) and (20), equation (22) can be written as

$$\phi_n(x,y)\left[\frac{D\omega_n^4}{\mu}T_n(t)+U_{n,tt}(t)\right]$$

$$=\frac{\phi_n(x,y)}{P^*}\sum_{q=1}^{\infty}\int_A \Big\{R_0\Big[\phi_{q,xx}(x,y)\phi_p(x,y)U_{q,tt}(t)+\phi_{q,yy}(x,y)\phi_p(x,y)U_{q,tt}(t)\Big]$$

$$-\frac{F_0}{\mu}\phi_q(x,y)\phi_p(x,y)U_q(t)+\frac{N_x}{\mu}\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)+\frac{N_y}{\mu}\phi_{q,yy}(x,y)\phi_p(x,y)U_q(t)$$

$$+\frac{Mg}{\mu}\phi_p(x,y)\delta(x-ct)\delta(y-s)-\frac{M}{\mu}\Big(\phi_q(x,y)\phi_p(x,y)U_{q,tt}(t)$$

$$+2c\phi_{q,x}(x,y)\phi_p(x,y)U_{q,t}(t)+c^2\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)\Big)\delta(x-ct)\delta(y-s)\Big]\Big\}dA \qquad (27)$$

Equation (27) implies that

$$U_{n,tt}(t)+\frac{D\omega_n^4}{\mu}U_n(t)$$

$$=\frac{1}{P^*}\sum_{q=1}^{\infty}\int_A \Big\{R_0\Big[\phi_{q,xx}(x,y)\phi_p(x,y)U_{q,tt}(t)+\phi_{q,yy}(x,y)\phi_p(x,y)U_{q,tt}(t)\Big]$$

$$+\frac{N_x}{\mu}\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)+\frac{N_y}{\mu}\phi_{q,yy}(x,y)\phi_p(x,y)U_q(t)$$

$$-\frac{F_0}{\mu}\phi_q(x,y)\phi_p(x,y)U_q(t)+\frac{Mg}{\mu}\phi_p(x,y)\delta(x-ct)\delta(y-s)$$

$$-\frac{M}{\mu}\Big(\phi_q(x,y)\phi_p(x,y)U_{q,tt}(t)+2c\phi_{q,x}(x,y)\phi_p(x,y)U_{q,t}(t)$$

$$+c^2\phi_{q,xx}(x,y)\phi_p(x,y)U_q(t)\Big)\delta(x-ct)\delta(y-s)\Big]\Big\}dA \qquad (28)$$

Equation (28) is a set of coupled second order ordinary differential equations. Expressing the Dirac-Delta function in the Fourier cosine series as [19]

$$\delta(x-c_i t)=\frac{1}{L_X}\left[1+2\sum_{j=1}^{\infty}\cos\frac{j\pi c_i t}{L_X}\cos\frac{j\pi x}{L_X}\right]$$

and

$$\delta(y-s)=\frac{1}{L_Y}\left[1+2\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}\cos\frac{k\pi y}{L_Y}\right] \qquad (29)$$

equation (28) then becomes

$$\frac{d^2U_n(t)}{dt^2}+\alpha_n^2U_n(t)-\frac{1}{P^*}\sum_{q=1}^{\infty}\Big\{R_0P_1^*\frac{d^2U_q(t)}{dt^2}-\frac{1}{\mu}\Big(F_0P_2^*-N_xk^0-N_yk^1\Big)U_q(t)$$

$$-\frac{M}{L_XL_Y\mu}\Bigg[2\Bigg(\frac{P_3^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)$$

$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\Bigg)\frac{d^2U_q(t)}{dt^2}+4c\Bigg(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)$$

$$+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\Bigg)\frac{dU_q(t)}{dt}$$

$$+2c^2\Bigg(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)$$

$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\Bigg)U_q(t)\Bigg]\Bigg\}=\frac{Mg}{P^*\mu}\phi_p(ct,s) \qquad (30)$$

where $\alpha_n^2=\frac{D\omega_n^4}{\mu}$,

$$k^0=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,\ k^1$$

$$=\int_0^{L_X}\int_0^{L_Y}\phi_{n,yy}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_1^*=k^0+k^1,\ P_2^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx$$

*and* $P_5^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$

Equation (30) is the transformed equation governing the problem of the rectangular plate on a Winkler elastic foundation.

Next, $\phi_n(x,y)$ are assumed to be the products of the beam functions $\psi_{ni}(x)$ and $\psi_{nj}(y)$ [20].
That is

$$\phi_n(x,y) = \psi_{ni}(x)\psi_{nj}(y) \qquad (31)$$

These beam functions can be defined respectively, as

$$\psi_{ni}(x) = \sin\frac{\Omega_{ni}x}{L_X} + A_{ni}\cos\frac{\Omega_{ni}x}{L_X} + B_{ni}\sinh\frac{\Omega_{ni}x}{L_X} + C_{ni}\cosh\frac{\Omega_{ni}x}{L_X} \qquad (32)$$

and

$$\psi_{nj}(y) = \sin\frac{\Omega_{nj}y}{L_Y} + A_{nj}\cos\frac{\Omega_{nj}y}{L_Y} + B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y} + C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y} \qquad (33)$$

where $A_{ni}$, $A_{nj}$, $B_{ni}$, $B_{nj}$, $C_{ni}$ and $C_{nj}$ are constants determined by the boundary conditions. $\Omega_{ni}$ and $\Omega_{nj}$ are called the mode frequencies.
Thus equation (30) becomes

$$\frac{d^2U_n(t)}{dt^2} + \alpha_n^2 U_n(t) - \frac{1}{P^*}\sum_{q=1}^{\infty}\Big\{R_0P_1^*\frac{d^2U_q(t)}{dt^2} - \frac{1}{\mu}\left(F_0P_2^* - N_xk^0 - N_yk^1\right)U_q(t)$$
$$-\Gamma\Bigg[2\left(\frac{P_3^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)$$
$$+4c\left(\frac{P_4^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)$$
$$+2c^2\left(\frac{P_5^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)$$
$$= \frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (34)$$

where $\Gamma = \dfrac{M}{L_X L_Y \mu}$.

Equation (34) is the fundamental equation of our problem. In what follows, we shall discuss two special cases of the equation (34) namely; the moving force and the moving mass problems.

## CASE I: MOVING FORCE PROBLEM

By setting $\Gamma = 0$ in equation (34), an approximate model of the differential equation describing the response of a rectangular plate resting on a Winkler elastic foundation and traversed by a moving force would be obtained.

Thus, setting $\Gamma = 0$ in equation (34), we have

$$\frac{d^2U_n(t)}{dt^2} + \alpha_n^2 T_n(t) - \frac{P_1^*R_0}{P^*}\sum_{q=1}^{\infty}\frac{d^2U_q(t)}{dt^2} + \frac{1}{\mu P^*}\left(F_0P_2^* - N_xk^0 - N_yk^1\right)\sum_{q=1}^{\infty}U_q(t)$$
$$= \frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (35)$$

Evidently, an exact analytical solution to this equation is not possible. Consequently, the approximate analytical solution technique, which is a modification of the asymptotic method of Struble coupled with the Laplace transform technique, is used as in [19], taking into account (19), (31), (32) and (33), to obtains

$$Z(x,y,t) = \sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty}\frac{K_0\Psi_{pj}(s)}{\gamma_{sf}[\gamma_{sf}^4 - \alpha_{pi}^4]}\Big\{[\gamma_{sf}^2 - \alpha_{pi}^2][C_{pi}\gamma_{sf}(\cosh\alpha_{pi}t - \cos\gamma_{sf}t)$$
$$+B_{pi}(\gamma_{sf}\sinh\alpha_{pi}t - \alpha_{pi}\sin\gamma_{sf}t)] + [\gamma_{sf}^2 + \alpha_{pi}^2][A_{pi}\gamma_{sf}(\cos\alpha_{pi}t - \cos\gamma_{sf}t)$$
$$-(\alpha_{pi}\sin\gamma_{sf}t - \gamma_{sf}\sin\alpha_{pi}t)]\Big\}[\sin\frac{\Omega_{ni}x}{L_X} + A_{ni}\cos\frac{\Omega_{ni}x}{L_X} + B_{ni}\sinh\frac{\Omega_{ni}x}{L_X} \qquad (36)$$
$$+C_{ni}\cosh\frac{\Omega_{ni}x}{L_X}][\sin\frac{\Omega_{nj}y}{L_Y} + A_{nj}\cos\frac{\Omega_{nj}y}{L_Y} + B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y} + C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y}]$$

where ${}_{sf}\gamma$ is the modified frequency corresponding to the frequency of the free system due to the presence of the axial forces $N_x$ and $N_y$. Equation (36)represents the transverse displacement response to a moving force of a rectangular plate resting on Winkler elastic foundation.

$$\Bigg[1 + \frac{2\varepsilon}{P^*}\left(\frac{P_2^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)\right.$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\Bigg]\frac{d^2U_n(t)}{dt^2} + \frac{4\varepsilon c}{P^*}\left(\frac{P_4^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)\right.$$
$$\left.+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dU_n(t)}{dt}$$
$$+\Bigg[\gamma_{sf}^2 + \frac{2\varepsilon c^2}{P^*}\left(\frac{P_5^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)\right.$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)\Bigg]U_n(t) + \frac{\varepsilon}{P^*}\sum_{\substack{q=1\\q\neq n}}\Bigg[2\left(\frac{P_2^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)\right.$$
$$\left.+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\frac{d^2U_q(t)}{dt^2}$$
$$4c\left(\frac{P_4^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)\right.$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dU_q(t)}{dt} + 2c^2\left(\frac{P_5^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)\right.$$
$$\left.+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)U_q(t)\Bigg]$$
$$= \frac{\varepsilon g L_X L_Y}{P^*}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (37)$$

where $\varepsilon = \dfrac{M}{L_X L_Y \mu}$ is the mass ratio.

We rearrange equation (37) to take the form

$$\frac{d^2U_n(t)}{dt^2} + \frac{\mu_0R_2(t)}{1+\mu_0R_1(t)}\frac{dU_n(t)}{dt} + \frac{\gamma_{sf}^2 + \mu_0R_3(t)}{1+\mu_0R_1(t)}U_n(t)$$
$$+\frac{\mu_0}{1+\mu_0R_1(t)}\sum_{\substack{q=1\\q\neq n}}^{\infty}\Big[R_1(t)\frac{d^2U_q(t)}{dt^2} + R_2(t)\frac{dU_q(t)}{dt}$$
$$+R_3(t)U_q(t)\Big] = \frac{\mu_0 g L_X L_Y}{[1+\mu_0R_1(t)]P^*}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (38)$$

where ε has been written as a function of the mass ratio $\mu_o$ and

$$R_1(t)=\frac{2}{P^*}\left[\frac{P_2^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right]$$

$$R_2(t)=\frac{2c}{P^*}\left[\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right]$$

$$R_3(t)=\frac{2c^2}{P^*}\left[\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right]$$

Equation (38) is solved using the same technique as in the previous case to obtain

$$Z(x,y,t)=\sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty}\frac{G_0\Psi_{pj}(s)}{\beta_{sf}[\beta_{sf}^4-\alpha_{pi}^4]}\{[\beta_{sf}^2-\alpha_{pi}^2][C_{pi}\beta_{sf}(\cosh\alpha_{pi}t-\cos\beta_{sf}t)+B_{pi}(\beta_{sf}\sinh\alpha_{pi}t-\alpha_{pi}\sin\beta_{sf}t)]+[\beta_{sf}^2+\alpha_{pi}^2][A_{pi}\beta_{sf}(\cos\alpha_{pi}t-\cos\beta_{sf}t)-(\alpha_{pi}\sin\beta_{sf}t-\beta_{sf}\sin\alpha_{pi}t)]\}[\sin\frac{\Omega_{ni}x}{L_X}+A_{ni}\cos\frac{\Omega_{ni}x}{L_X}+B_{ni}\sinh\frac{\Omega_{ni}x}{L_X}+C_{ni}\cosh\frac{\Omega_{ni}x}{L_X}][\sin\frac{\Omega_{nj}y}{L_Y}+A_{nj}\cos\frac{\Omega_{nj}y}{L_Y}+B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y}+C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y}] \quad (39)$$

where $G_0=\frac{\mu_0 g L_X L_Y}{P^*}$

and $$\beta_{sf}=\gamma_{sf}\left[1-\frac{\mu_0}{2}\left(R_1-\frac{R_3}{\gamma_{sf}^2}\right)\right] \quad (40)$$

is the modified frequency corresponding to the frequency of the free system due to the presence of the moving mass.

Equation (39) is the transverse displacement response to a moving mass of a rectangular plate resting on Winkler elastic foundation. The constants $A_{ni}$, $A_{pi}$, $A_{nj}$, $A_{pj}$, $B_{ni}$, $B_{pi}$, $B_{nj}$, $B_{pj}$, $C_{ni}$, $C_{pi}$, $C_{nj}$ and $C_{pj}$ are to be determined from the choice of the end support condition.

## 4 Analysis of the Solution

The phenomenon of resonance is examined in this section. Equation (36) clearly shows that the rectangular plate on a Winkler elastic foundation and traversed by a moving force reaches a state of resonance whenever These results hold for other choices of elastically supported ends such as clamped-elastic and elastic-free ends conditions.

$$\gamma_{sf}=\frac{\Omega_{pi}c}{L_X} \quad (41)$$

while equation (39) shows that the same plate under the action of a moving mass experiences resonance effect whenever

$$\beta_{sf}=\frac{\Omega_{pi}c}{L_X} \quad (42)$$

Equations (40) and (42) imply that

$$\beta_{sf}=\gamma_{sf}\left[1-\frac{\mu_0}{2}\left(R_1-\frac{R_3}{\gamma_{sf}^2}\right)\right]=\frac{\Omega_{pi}c}{L_X} \quad (43)$$

Consequently from equations (41) and (43), for the same natural frequency of the plate, the resonance is reached earlier when we consider the moving mass system than when we consider the moving force system.

## 5. Illustrative Examples

a. Simple-Elastic Rectangular plate.

At x = 0 and x = LX, the plate is taken to be simply supported and at the edges y = 0 and y = LY, it is taken to be elastically supported.

Using the conditions (2-9) in equations (32) and (33), the following values of the constants and the frequency equation are obtained for the elastic edges.

$$C_{nj}=\frac{\left[\frac{\Omega_{nj}}{L_Y}-k_1r_2\right]\sin\Omega_{nj}+\left[k_1+\frac{r_2\Omega_{nj}}{L_Y}\right]\cos\Omega_{nj}-\frac{r_1\Omega_{nj}}{L_Y}\sinh\Omega_{nj}+k_1r_1\cosh\Omega_{nj}}{k_1r_1\sin\Omega_{nj}-\frac{r_1\Omega_{nj}}{L_Y}\cos\Omega_{nj}+\left[\frac{r_3\Omega_{nj}}{L_Y}-k_1\right]\sinh\Omega_{nj}+\left[\frac{\Omega_{nj}}{L_Y}-k_1r_3\right]\cosh\Omega_{nj}}=\frac{-\left[\frac{r_2\Omega_{nj}^3}{L_Y^3}+k_2\right]\sin\Omega_{nj}+\left[\frac{\Omega_{nj}^3}{L_Y^3}-k_2r_2\right]\cos\Omega_{nj}-k_2r_1\sinh\Omega_{nj}-\frac{r_1\Omega_{nj}^3}{L_Y^3}\cosh\Omega_{nj}}{\frac{r_1\Omega_{nj}^3}{L_Y^3}\sin\Omega_{nj}+k_2r_1\cos\Omega_{nj}+\left[\frac{\Omega_{nj}^3}{L_Y^3}+k_2r_3\right]\sinh\Omega_{nj}+\left[\frac{r_3\Omega_{nj}^3}{L_Y^3}+k_2\right]\cosh\Omega_{nj}}, \quad (44)$$

$$A_{nj}=r_1C_{nj}+r_2 \quad and \quad B_{nj}=r_3C_{nj}+r_1 \quad (45)$$

$$r_1=\frac{\frac{\Omega_{nj}^4}{L_Y^4}+k_1k_2}{\frac{\Omega_{nj}^4}{L_Y^4}-k_1k_2}; \quad r_2=\frac{-\frac{2k_1\Omega_{nj}^3}{L_Y^3}}{\frac{\Omega_{nj}^4}{L_Y^4}-k_1k_2} \quad and \quad r_3=\frac{\frac{-2k_2\Omega_{nj}}{L_Y}}{\frac{\Omega_{nj}^4}{L_Y^4}-k_1k_2}.$$

Equation (44) when simplified yields

$$\tan\Omega_{nj}=\tanh\Omega_{nj} \quad (46)$$

which is termed the frequency equation for the elastic edge, such that

$$\Omega_1=3.927,\ \Omega_2=7.069,\ \Omega_3=10.210,\ ... \quad (47)$$

For the simple edges, it can be shown that

$A_{ni}=0, B_{ni}=0, C_{ni}=0$, and $_{ni}=n_i$ (48)

Similarly, $A_{pi}=0, B_{pi}=0, C_{pi}=0$, and $_{pi}=p_i$ (49)

Using (44), (45), (47), (48) and (49) in equations (36) and (39) one obtains the displacement response respectively to a moving force and a moving mass of a simple-elastic prestressed rectangular plate resting on a constant Winkler elastic foundation.

**vb. Elastic support at all edges.**

Using the conditions (10-17) in equations (32) and (33), one obtains

$$C_{ni} = \frac{\left[\frac{\Omega_{ni}}{L_X} - k_1 r_2(i)\right]\sin\Omega_{ni} + \left[k_1 + \frac{r_2(i)\Omega_{ni}}{L_X}\right]\cos\Omega_{ni} - \frac{r_1(i)\Omega_{ni}}{L_X}\sinh\Omega_{ni} + k_1 r_1(i)\cosh\Omega_{ni}}{k_1 r_1(i)\sin\Omega_{ni} - \frac{r_1(i)\Omega_{ni}}{L_X}\cos\Omega_{ni} + \left[\frac{r_3(i)\Omega_{ni}}{L_X} - k_1\right]\sinh\Omega_{ni} + \left[\frac{\Omega_{ni}}{L_X} - k_1 r_3(i)\right]\cosh\Omega_{ni}}$$

$$= \frac{-\left[\frac{r_2(i)\Omega_{ni}^3}{L_X^3} + k_2\right]\sin\Omega_{ni} + \left[\frac{\Omega_{ni}^3}{L_X^3} - k_2 r_2(i)\right]\cos\Omega_{ni} - k_2 r_1(i)\sinh\Omega_{ni} - \frac{r_1(i)\Omega_{ni}^3}{L_X^3}\cosh\Omega_{ni}}{\frac{r_1(i)\Omega_{ni}^3}{L_X^3}\sin\Omega_{ni} + k_2 r_1(i)\cos\Omega_{ni} + \left[\frac{\Omega_{ni}^3}{L_X^3} + k_2 r_3(i)\right]\sinh\Omega_{ni} + \left[\frac{r_3(i)\Omega_{ni}^3}{L_X^3} + k_2\right]\cosh\Omega_{ni}},$$

(

$$A_{ni} = r_1(i)C_{ni} + r_2(i) \;\; and \;\; B_{ni} = r_3(i)C_{ni} + r_1(i) \qquad (51)$$

where

$$r_1(i) = \frac{\frac{\Omega_{ni}^4}{L_X^4} + k_1 k_2}{\frac{\Omega_{ni}^4}{L_X^4} - k_1 k_2}; \quad r_2(i) = \frac{-\frac{2k_1\Omega_{ni}^3}{L_X^3}}{\frac{\Omega_{ni}^4}{L_X^4} - k_1 k_2} \;\; and \;\; r_3(i) = \frac{\frac{-2k_2\Omega_{ni}}{L_X}}{\frac{\Omega_{ni}^4}{L_X^4} - k_1 k_2}.$$

$$C_{nj} = \frac{\left[\frac{\Omega_{nj}}{L_Y} - k_1 r_2(j)\right]\sin\Omega_{nj} + \left[k_1 + \frac{r_2(j)\Omega_{nj}}{L_Y}\right]\cos\Omega_{nj} - \frac{r_1(j)\Omega_{nj}}{L_Y}\sinh\Omega_{nj} + k_1 r_1(j)\cosh\Omega_{nj}}{k_1 r_1(j)\sin\Omega_{nj} - \frac{r_1(j)\Omega_{nj}}{L_Y}\cos\Omega_{nj} + \left[\frac{r_3(j)\Omega_{nj}}{L_Y} - k_1\right]\sinh\Omega_{nj} + \left[\frac{\Omega_{nj}}{L_Y} - k_1 r_3(j)\right]\cosh\Omega_{nj}}$$

$$= \frac{-\left[\frac{r_2(j)\Omega_{nj}^3}{L_Y^3} + k_2\right]\sin\Omega_{nj} + \left[\frac{\Omega_{nj}^3}{L_Y^3} - k_2 r_2(j)\right]\cos\Omega_{nj} - k_2 r_1(j)\sinh\Omega_{nj} - \frac{r_1(j)\Omega_{nj}^3}{L_Y^3}\cosh\Omega_{nj}}{\frac{r_1(j)\Omega_{nj}^3}{L_Y^3}\sin\Omega_{nj} + k_2 r_1(j)\cos\Omega_{nj} + \left[\frac{\Omega_{nj}^3}{L_Y^3} + k_2 r_3(j)\right]\sinh\Omega_{nj} + \left[\frac{r_3(j)\Omega_{nj}^3}{L_Y^3} + k_2\right]\cosh\Omega_{nj}}, \qquad (5$$

$$A_{nj} = r_1(j)C_{nj} + r_2(j) \;\; and \;\; B_{nj} = r_3(j)C_{nj} + r_1(j) \qquad (53)$$

where

$$r_1(j) = \frac{\frac{\Omega_{nj}^4}{L_Y^4} + k_1 k_2}{\frac{\Omega_{nj}^4}{L_Y^4} - k_1 k_2}; \quad r_2(j) = \frac{-\frac{2k_1\Omega_{nj}^3}{L_Y^3}}{\frac{\Omega_{nj}^4}{L_Y^4} - k_1 k_2} \;\; and \;\; r_3(j) = \frac{\frac{-2k_2\Omega_{nj}}{L_Y}}{\frac{\Omega_{nj}^4}{L_Y^4} - k_1 k_2}.$$

Equations (50) and (52) when simplified yield

$$\tan\Omega_{ni} = \tanh\Omega_{ni} \quad and \quad \tan\Omega_{nj} = \tanh\Omega_{nj} \qquad (54)$$

Using (50), (51), (52), (53), and (54) in equations (36) and (39) one obtains the transverse-displacement response respectively to a moving force and a moving mass of an elastically supported and prestressed rectangular plate resting on a constant Winkler elastic foundation.

## 6 Numerical Calculations and Discussion of Results

For the calculations of practical interests in dynamics of structures, a rectangular plate of length $L_Y$ = 0.914m and breadth $L_X$ = 0.457m is considered. The velocity of the mass is assumed to be 0.8123m/s and the values for E, S and are chosen to be 3.109x10$^9$kg/m$^2$, 0.4m and 0.2 respectively. Figures 6.1 and 6.2 show the effect of axial forces $N_x$ and $N_y$ respectively on the transverse deflection of the simple-elastic prestressed rectangular plate for the case of moving mass. The graphs show that the response amplitudes decrease as the values of $N_x$ and $N_y$ increase.

The influence of $N_x$ and $N_y$ on the transverse deflection of moving mass displayed in Figures 6.3 and 6.4 respectively show that an increase in the value of each of $N_x$ and $N_y$ decreases the deflection of the elastically supported (elastic-elastic) prestressed rectangular plate resting on constant Winkler elastic foundation. Figure 6.5 displays the deflection of moving mass of the prestressed and elastically supported (elastic-elastic) rectangular plate for various values of rotatory inertia correction factor $R_0$, it is shown that as $R_0$ increases, the response amplitudes of the plate decrease.

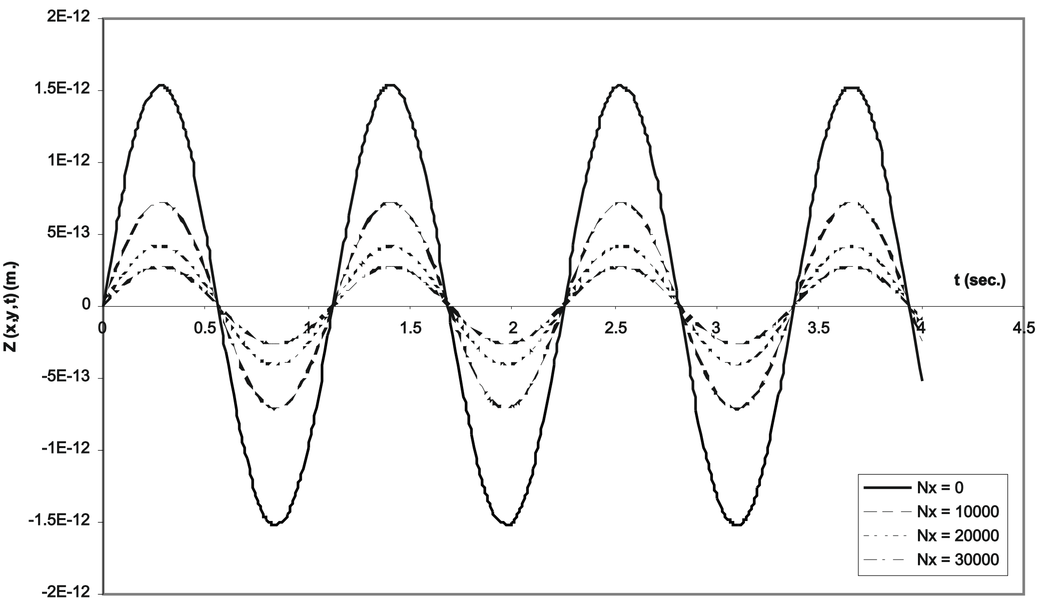


**Fig.6.1: Deflection of simple-elastic plate on Winkler elastic foundation and traversed by moving mass for various values of axial force Nx.**

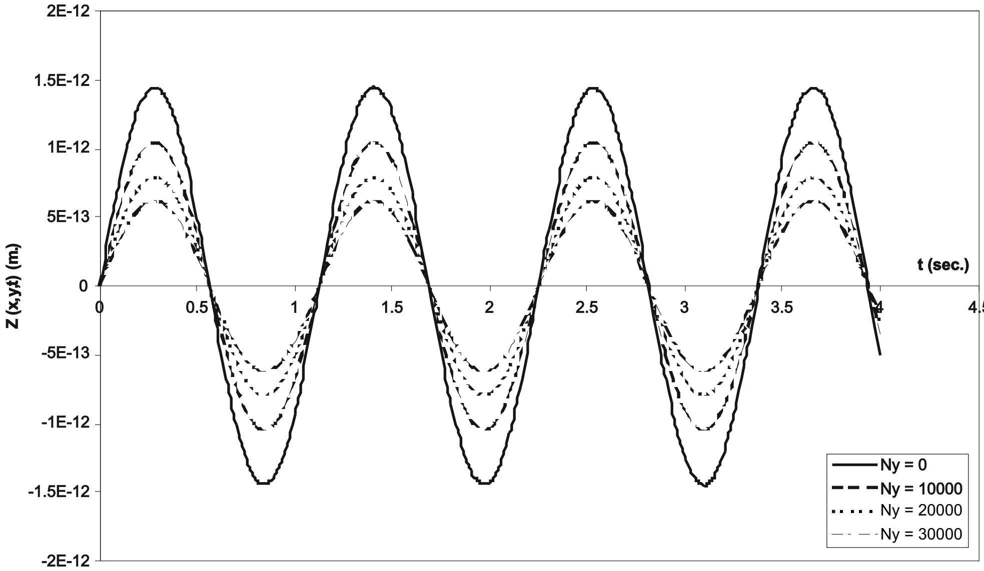


**Fig. 6.2: Deflection of simple-elastic plate on Winkler foundation and traversed by moving mass for various values of axial force Ny.**

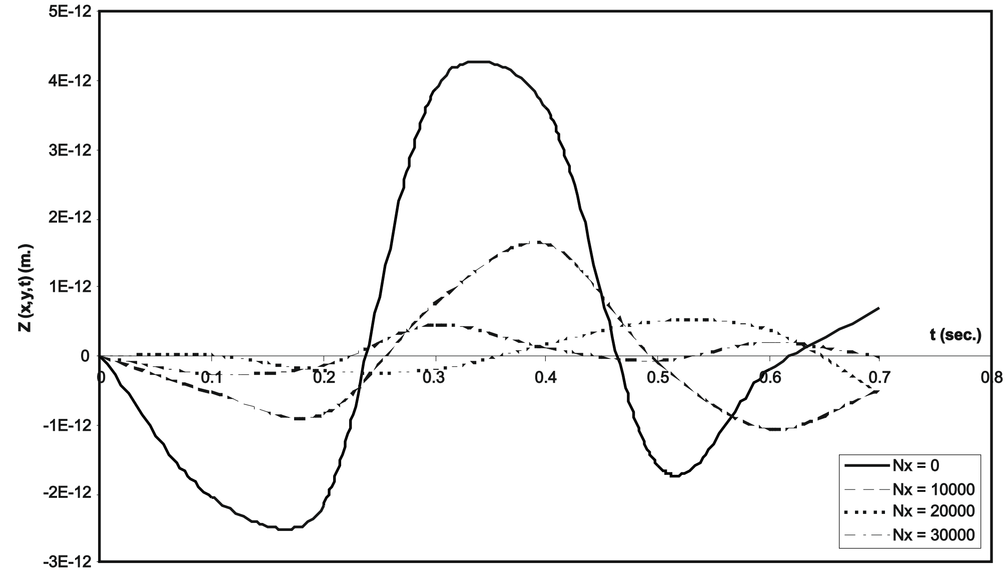


**Fig. 6.3: Displacement profile of elastic-elastic plate on Winkler foundation and traversed by moving mass for various values of axial force Nx.**

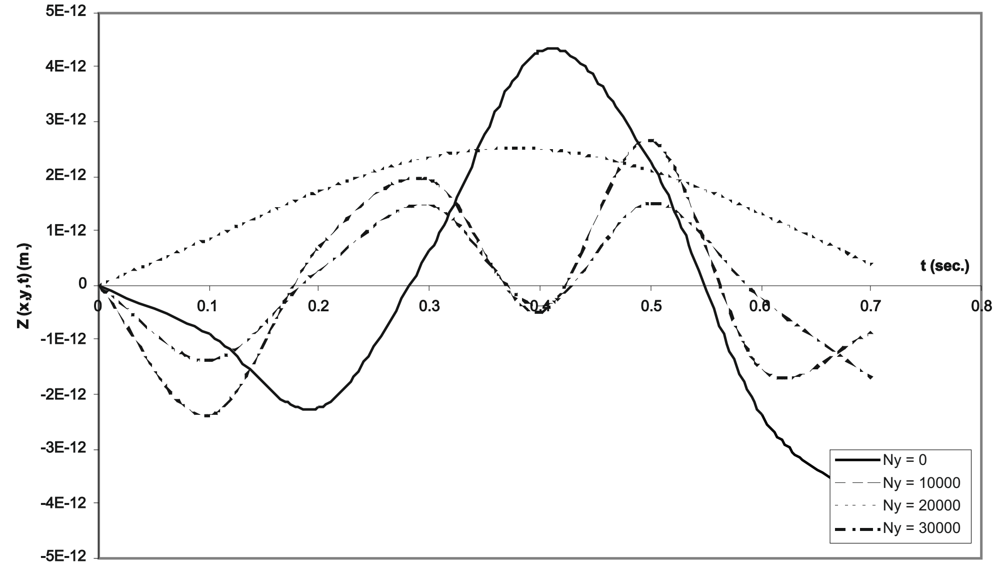


**Fig. 6.4: Displacement profile of elasti-elastic plate on Winkler elastic foundation and traversed by moving mass for various values of axial force Ny.**

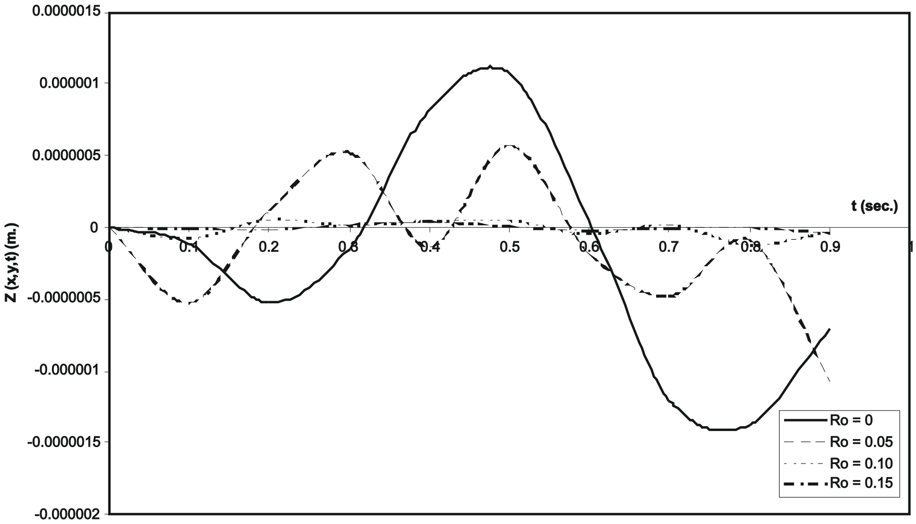


**Fig. 6.5: Deflection of elastic-elastic plate resting on Winkler elastic foundation and traversed by moving mass for various values of rotatory inertia Ro.**

The comparison of the displacement curves of moving force and moving mass of an elastic-elastic prestressed rectangular plate for fixed values of $N_x$ and $N_y$ displayed in Figure 6.6 shows that the response amplitude of a moving mass is greater than that of a moving force.

Figure 6.7 displays the effect of the mass ratio on the transverse displacement curves of the moving mass for elastic-elastic prestressed rectangular plate for fixed values of $N_x$ and $N_y$, the response amplitude increases as the mass ratio increases and approaches the deflection of moving force as the mass ratio approaches zero.

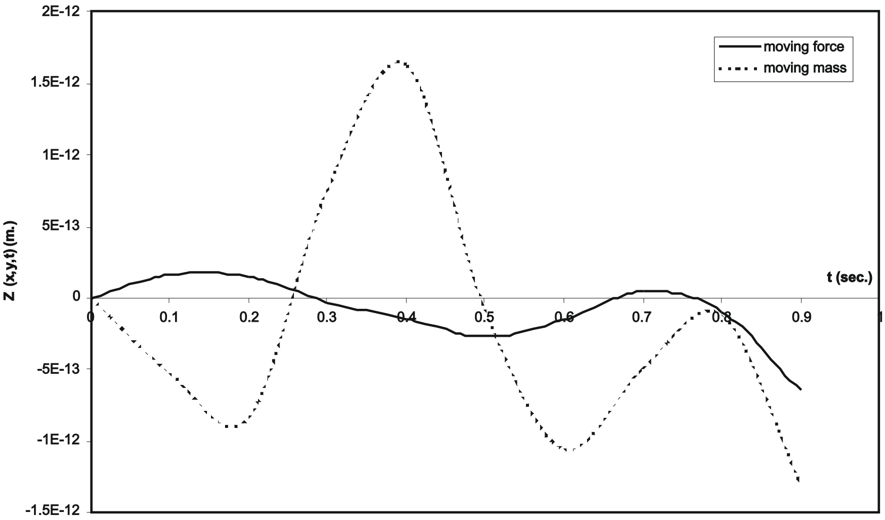


**Fig. 6.6: Comparison of moving force and moving mass cases of elastic-elastic rectangular plate resting on Winkler elastic foundation.**

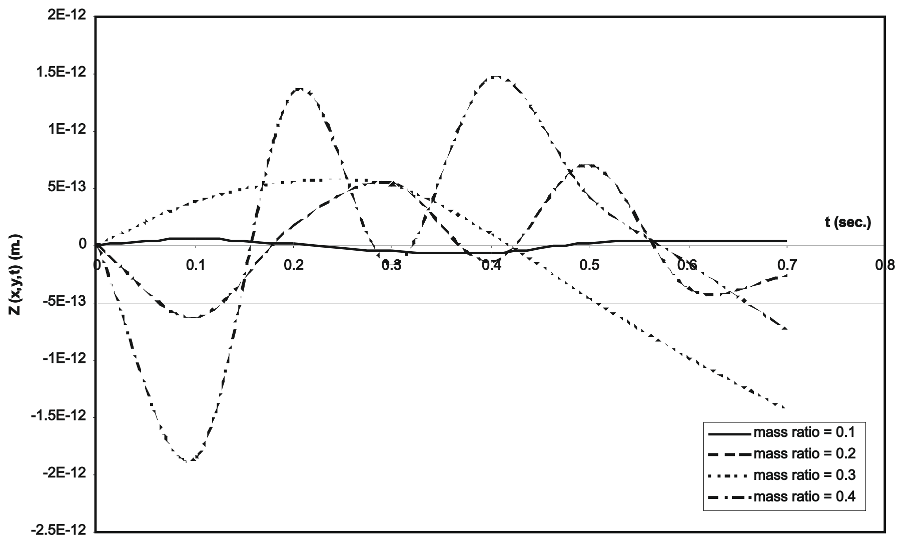


**Fig. 6.7: Deflection of moving mass of elastic-elastic rectangular plate resting on Winkler elastic foundation for various values of mass ratio.**

These results hold for other choices of elastically supported ends such as clamped-elastic and elastic-free ends conditions.

## 7 Conclusion

The dynamic behaviours under moving concentrated masses of prestressed and elastically supported rectangular plates resting on constant Winkler elastic foundation is considered in this work. The method based on Separation of variables is used to transform the governing equation to a set of coupled second order ordinary differential equations. The modified Struble's technique and the method of integral transformations are employed to obtain the closed form solution of the transformed equation for both cases of moving force and moving mass problems.

From the analyses of the solutions, for the same natural frequency, the critical speed (and the natural frequency) for the system of the prestressed and elastically supported rectangular plate traversed by a moving mass is smaller than that of the same system traversed by a moving force. Thus, for the same natural frequency of the plate, the resonance is reached earlier when we consider the moving mass system than when we consider the moving force system. The analyses show that the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

The results in plotted curves show that as each of the axial forces $N_x$ and $N_y$ increases, the response amplitudes of the plates decrease for both cases of moving force and moving mass problems for all the non-classical boundary conditions considered. Also, the response amplitudes of the prestressed and elastically supported plate decrease as the value of the rotatory inertia correction factor increases.

Furthermore, the response amplitude for the moving mass problem is greater than that of the moving force problem for fixed values of $N_x$ and $N_y$, this implies that resonance is reached earlier in moving mass problem than in moving force problem, it is therefore unsafe to rely on the moving force solutions. Also, as the mass ratio $\Gamma$ approaches zero, the response amplitudes of the moving mass problem approach that of the moving force problem of the prestressed and elastically supported rectangular plate resting on constant Winkler elastic foundation for the illustrative examples of the non-classical boundary conditions considered.

Finally, the results in this work agree with what obtain in literature [2, 6, 19, 21]. Hence the method employed in this work is accurate and the solutions are convergent.

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