FEDERAL UNIVERSITY OF TECHNOLOGY, AKURE
JOURNAL OF ENGINEERING AND ENGINEERING TECHNOLOGY

DAMPING EFFECTS ON THE DYNAMIC BEHAVIOUR OF BEAM-TYPE STRUCTURE WITH CLASSICAL BOUNDARY CONDITIONS UNDER MOVING DISTRIBUTED LOAD

[1]Famuagun, K. S. and [2]Oni S. T.
[1]Department of Mathematics Adeyemi Federal University of Education, Ondo, Ondo State, Nigeria
[2]Department of Mathematical Sciences, Federal University of Technology, Akure, Ondo State, Nigeria

**Corresponding author: famuagunks@afued.edu.ng*

Famuagun K. S. and Oni S. T. (2026): Damping Effects on the Dynamic Behaviour of Beam-type Structure with Classical Boundary Conditions under Moving Distributed Load. FUTA Journal of Engineering and Engineering Technology 20(2), 1-10.

Received Date: 31.03.2026 **Accepted Date: 12.05.2026**

Abstract
This paper investigates damping effects on the dynamic behaviour of beam-type structure with classical boundary conditions. It is an extension of the work of Famuagun (2023). The problem is governed by a fifth-order partial differential equation, which is reduced to second-order, coupled ordinary differential equations through the finite generalized integral transform method. The analytical solution procedures involved Heaviside function expansion, a modification of the Struble asymptotic technique, and Laplace transformation. Numerical computations depict that both damping coefficients incorporated into the governing system, significantly reducing the displacement amplitude of the beam. However, damping due to resistance to transverse displacement possesses stronger effects compared to damping due to strain velocity. In addition, resonance is observed to occur earlier in the moving mass case than in the moving force case. Finally, the results illustrate the importance of damping parameters in influencing vibration of structural members.

Keywords: *Dynamic Behaviour, Distributed Masses, Damped Beams, Classical Boundary Conditions*

Introduction

The analysis of structural elements such as rod, beam, plate and shell under the influence of an external force (like beam-type structure under moving loads) is a cornerstone of modern engineering, especially in civil engineering, underpinning the safe and efficient design of critical infrastructure such as overhead bridges, slab of a tower and railways.

Accurate prediction of the dynamic response of structures to distributed load action is a fundamental challenge that varies in both space and time. In classical studies, these loads were idealized as moving forces, which implies that the inertial effect of the moving mass is not considered whereas the moving mass case in reality provides a more accurate representation. One of the earliest works in this field of study was the seminal work of Timoshenko which laid the foundation for many studies in this field, and subsequent studies have expanded upon it to employ more realistic structural features and loading with boundary conditions (Timoshenko, 1992).

A good number of literatures have focused on the influence of boundary conditions, foundation stiffness, and structural prestress. Examples of such in literature are the studies of Adams (1995), who examined the critical speeds of tensioned beams on elastic foundations, while Oni and Omolofe (2011) extended this to prestressed Rayleigh beams under moving masses. Wu and Gao (2015) also demonstrated in their work on double-beam systems under harmonic loads, this increases the complexity of the system since it involves multiple structural elements. However, no damping has been designed into their structural systems. Nonetheless, the excitative force on the structural systems with low or no damping may be greatly amplified by resonance, thus leading to fatigue-inducing conditions, therefore, the inclusion of resonance in the structural system must be accompanied by high damping.

In the study of vibration or oscillatory system, damping is the process through which vibrational energy is dissipated, is a critical parameter in controlling the resonant amplitude of a structure. To this end, Crandall (1970) made known that primarily, damping is an essential parameter in controlling vibration response amplitude under conditions of steady state resonance and random excitement. Despite its importance, damping is treated in a simplistic manner. Oliveto and Greco (2002) studied how modal damping ratio vary with boundary conditions by addressing the characterization of damping in dynamical systems yet with all these, they did not use an analytic solutions approach to explain the response of the beam structure under moving load. Lan et al. (2016), in a more recent study examine the interactions between the static axial load, structural damping and moving load but the load was moving with constant amplitude with a constant velocity and the analyses largely relied on numerical simulations.

Recent studies in this area are the works of Zhao et al. (2023), Zhang et al. (2023) and Li et al. (2024) who investigated damping effects analytically, in beam-type structures under moving inertial loads and established that damping parameters strongly influence critical velocity and resonance behaviour. Despite these advancements, closed-form analytical solutions for damped beam-type structures under moving distributed masses with classical boundary conditions remain limited.

In all the studies involving damping effects on structured vibrations, numerical simulations are mostly used whereas, closed form solutions provide more vital information on the vibrating systems. Therefore, in this study damping effects on the dynamic behaviour of beam-type structure with classical boundary conditions under a moving distribution load is considered using closed-form solution approach.

Mathematical Model

A uniform beam of length L, mass per unit length $\overline{m}$, and flexural rigidity EI was considered. The beam rests on a two-parameter elastic foundation characterized by a stiffness constant K and a shear modulus G. A distributed load of mass M per unit length moves across the beam at a constant velocity u. The load is assumed to always be in full contact with the structure. The equation of motion for the transverse displacement V(x,t) is derived by considering the dynamic equilibrium of the beam element. Incorporation of the effects of damping and the inertia of the moving mass leads to the following governing equation (Famuagun, 2023):

$$\mathrm{EI}\frac{\partial^2 \mathrm{V}(x,\mathrm{t})}{\partial \mathrm{t}^2} - C_s\mathrm{I}\,\frac{\partial^5 V(x,\mathrm{t})}{\partial x^4\,\partial \mathrm{t}} + \overline{m}\,\frac{\partial^2 V(x,\mathrm{t})}{\partial \mathrm{t}^2} + C\,\frac{\partial \mathrm{V}(x,\mathrm{t})}{\partial \mathrm{t}} - \overline{m}\mathrm{R}^0\,\frac{\partial^4 \mathrm{V}(x,\mathrm{t})}{\partial x,^2\,\partial t^2} - N\,\frac{\partial^2 V(x,\mathrm{t})}{\partial \mathrm{t}^2} - K\mathrm{V}(x,\mathrm{t}) - \frac{G\partial^2 V(x,t)}{\partial x^2} + \mathrm{M}H(x-\mathrm{u}t)\left[\frac{\partial^2 V(x,t)}{\partial t^2} + 2\mathrm{u}\,\frac{\partial^2 V(x,t)}{\partial x^2} + u^2\,\frac{\partial^2 V(x,t)}{\partial \mathrm{x}^2}\right] = \mathrm{M}gH(x-\mathrm{u}t) \qquad (1)$$

The initial conditions are

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial \mathrm{t}} \qquad (2)$$

while the boundary conditions at x = 0 and x = L are arbitrary at this stage and will be specified later for the classical cases.

where

Cs: damping due to resistance to transverse displacement

C: damping due to resistance to strain velocity.

N: axial forces

$\delta(.)$ is the Dirac delta function and other parameters are presented in the table of nomenclatures.

Solution Procedure

We employ a generalized integral transform technique to solve eq. (1). The solution is expressed as an infinite series of the beam's normal modes which satisfy the given boundary conditions.

$$\overline{\mathrm{V}}_m(\mathrm{t}) = \int_0^1 \mathrm{V}(x,\mathrm{t})U_{\mathrm{m}}(x)\mathrm{dx} \qquad (3)$$

$$\mathrm{V(x,t)} = \sum_{\mathrm{m}=1}^{\infty} \overline{V}_m\,(\mathrm{t})\,\frac{U_{\mathrm{m}}(x)}{N_m} \qquad (4)$$

Where the normalization constant is given by

$$N_m = \int_0^1 mU_m^2(x)\mathrm{d}x \quad (5)$$

For a uniform beam, the eigenfunction are:

$$.U_m(x) = sin\frac{\lambda_m x}{L} + A_m cos\frac{\lambda_m x}{L} + B_m sinh\frac{\lambda_m x}{L} + C_m cosh\frac{\lambda_m x}{L} \quad \text{(6a)}$$

Where λ_m are the eigenvalues (frequency parameters) determined from the boundary conditions and A_m, B_m and C_m are constant and derived from the same conditions.

Using (3) and (4) in (1) and using the orthogonality properties of the eigenfunctions transforms the partial differential equation into a set of coupled, second-order ordinary differential equations of the form:

$$\bar{V}_{\mathrm{tt}}(m,\mathrm{t}) + (\omega_m^2 + \frac{K}{\bar{\mathrm{m}}})\,\bar{\mathrm{V}}(\mathrm{m},\mathrm{t}) - \frac{\mathrm{R}^0}{\mathrm{L}}[\mathrm{L}B_A(k,\mathrm{m})\bar{\mathrm{V}}_{tt}(k,\mathrm{t}) - \frac{\mathrm{L}}{\mathrm{R}^0}\left(\frac{\mathrm{C}}{\bar{m}} - \frac{\omega_m^2}{\mathrm{EI}}\right)\bar{\mathrm{v}}_t(m,\mathrm{t}) - \frac{N}{\bar{m}L\mathrm{R}^0}\sum_{\mathrm{k}=1}^{\infty}\mathrm{LB}_B(k,\mathrm{m})\bar{\mathrm{V}}(m,\mathrm{t}) - \frac{G}{\bar{m}L\mathrm{R}^0}\sum_{\mathrm{k}=1}^{\infty}\mathrm{LB}_B(k,\mathrm{m})\bar{\mathrm{V}}(m,\mathrm{t})\Bigg] +$$

$$\mu^0\Bigg\{\sum_{\mathrm{k}=1}^{\infty}\mathrm{L}\left[\frac{1}{4}\bar{\mathrm{V}}_{tt}(\mathrm{k},\mathrm{t})B_C(\mathrm{k},\mathrm{m}) + \frac{1}{\pi}\sum_{\mathrm{n}=1}^{\infty}\frac{cos(2n+1)\pi\mathrm{ut}}{2\mathrm{n}+1}\bar{\mathrm{V}}_{tt}(\mathrm{k},\mathrm{t})\,B_D(n,\mathrm{k},\mathrm{m}) - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{sin(2n+1)\pi\mathrm{ut}}{2n+1}\bar{\mathrm{V}}_{tt}(\mathrm{k},\mathrm{t})B_E(n,\mathrm{k},\mathrm{m}) + \frac{\mathrm{u}}{2}\bar{\mathrm{V}}_t(\mathrm{k},\mathrm{t})B_F(\mathrm{k},\mathrm{m}) + \frac{\mathrm{u}^2}{4}\bar{V}(\mathrm{k},\mathrm{t})B_I(\mathrm{k},\mathrm{m}) + \frac{2\mathrm{u}}{\pi}\sum_{\mathrm{n}=1}^{\infty}\frac{cos(2n+1)\pi\mathrm{ut}}{2n,+1}\bar{\mathrm{V}}_t(\mathrm{k},\mathrm{t})\,B_G(n,\mathrm{k},\mathrm{m}) - \frac{2\mathrm{u}}{\pi}\sum_{n=1}^{\infty}\frac{sin(2n+1)\pi\mathrm{ut}}{2n+1}\bar{\mathrm{V}}_t(\mathrm{k},\mathrm{t})B_H(n,\mathrm{k},\mathrm{m}) + \frac{\mathrm{u}^2}{\pi}\sum_{\mathrm{n}=1}^{\infty}\frac{cos(2n+1)\pi\mathrm{ut}}{2n,+1}\bar{V}(\mathrm{k},\mathrm{t})\,B_J(n,\mathrm{k},\mathrm{m}) - \frac{\mathrm{u}^2}{\pi}\sum_{n=1}^{\infty}\frac{sin(2n+1)\pi\mathrm{ut}}{2n+1}\bar{V}(\mathrm{k},\mathrm{t})\,B_k(n,\mathrm{k},\mathrm{m})\right\}$$

$$= \mathrm{M}g\frac{L}{\bar{m}\lambda_m}[-cos\lambda_m + A_m sin\lambda_m + B_m cosh\lambda_m + C_m sinh\lambda_m + cos\frac{\lambda_m\mathrm{ut}}{L} + A_m sin\frac{\lambda_m\mathrm{ut}}{\mathrm{L}} + B_m sinh\frac{\lambda_m\mathrm{ut}}{\mathrm{L}} + C_m cosh\frac{\lambda_m\mathrm{ut}}{\mathrm{L}}] \quad \text{(6b)}$$

Where

$$\mu^0 = \frac{\mathrm{M}}{\bar{m}L} \quad (7)$$

From (6b), two cases emerged, **case I** (**moving force problem)**, the inertia terms are neglected i.e $\mu^0 = 0$. The resulting transformed equation, after some algebraic manipulation can be written as

$$\bar{V}_{\mathrm{tt}}(m,\mathrm{t}) + (\omega_m^2 + \frac{K}{\bar{\mathrm{m}}})\,\bar{\mathrm{V}}(\mathrm{m},\mathrm{t}) - \frac{\mathrm{R}^0}{\mathrm{L}}[\mathrm{L}B_A(k,\mathrm{m})\bar{\mathrm{V}}_{tt}(k,\mathrm{t}) - \frac{\mathrm{L}}{\mathrm{R}^0}\left(\frac{\mathrm{C}}{\bar{m}} - \frac{\omega_m^2}{\mathrm{EI}}\right)\bar{\mathrm{v}}_t(m,\mathrm{t}) - \frac{N}{\bar{m}L\mathrm{R}^0}\sum_{\mathrm{k}=1}^{\infty}\mathrm{LB}_B(k,\mathrm{m})\bar{\mathrm{V}}(m,\mathrm{t}) - \frac{G}{\bar{m}L\mathrm{R}^0}\sum_{\mathrm{k}=1}^{\infty}\mathrm{LB}_B(k,\mathrm{m})\bar{\mathrm{V}}(m,\mathrm{t})\Bigg]$$

$$= \mathrm{M}g\frac{L}{\bar{m}\lambda_m}[-cos\lambda_m + A_m sin\lambda_m + B_m cosh\lambda_m + C_m sinh\lambda_m + cos\frac{\lambda_m\mathrm{ut}}{L} + A_m sin\frac{\lambda_m\mathrm{ut}}{\mathrm{L}} + B_m sinh\frac{\lambda_m\mathrm{ut}}{\mathrm{L}} + C_m cosh\frac{\lambda_m\mathrm{ut}}{\mathrm{L}}] \quad (8)$$

To obtain an approximate analytic solution, we employ a modification of Struble's asymptotic method. For small parameter $\varepsilon = \frac{\varepsilon_0}{1+\varepsilon_0}$, the homogeneous part of the transformed equation is solved, leading to an effective frequency that accounts for the load inertia. After applying Laplace transforms and convolution theory, the displacement response is expressed in closed form. For the moving force case, the solution is

$$\mathrm{V}(x,\mathrm{t}) = \sum_{\mathrm{m}=1}^{\infty}\frac{\wp_{bj}^0}{R(\mathrm{x})(\varpi^4 - \dot{\omega}_{bj}^4)}\Bigg[\frac{G_F}{\dot{\omega}_{bj}}(\varpi^4 - \dot{\omega}_{bj}^4)(1 - cos\dot{\omega}_{bj}) + (\varpi^2 - \dot{\omega}_{bj}^2)(cos\dot{\omega}_{bj}^2 t - cos\varpi\mathrm{t} - A_m\frac{\varpi\sin\dot{\omega}_{bj}}{\dot{\omega}_{bj}} + A_m sin\varpi\mathrm{t}) - (\varpi^2 - \dot{\omega}_{bj}^2)(cosh\varpi\mathrm{t} - B_m cos\dot{\omega}_{bj}\mathrm{t} - C_m sinh\varpi\mathrm{t}\Big]$$

$$-C_m \frac{1}{\dot{\omega}_{bj}} sin\dot{\omega}_{bj}\mathrm{t})$$

$$\left[sin\frac{\lambda_m x}{\mathrm{L}} + A_m cos\frac{\lambda_m x}{\mathrm{L}} + B_m sinh\frac{\lambda_m x}{\mathrm{L}} + C_m cosh\frac{\lambda_m x}{\mathrm{L}}\right] \quad (9)$$

Where $\varpi = \frac{\lambda\pi \mathrm{u}}{\mathrm{L}}$ (10)

Expression (9) represents the solution to the moving forces solution.

For the **moving mass case**, a similar expression is obtained, with $\dot{\omega}_{bj}$replace by a modified frequency σ_{bj} that includes contributions from the moving mass inertia. The solution is

$$\mathrm{V}(x,\mathrm{t}) = \sum_{m=1}^{\infty} \frac{\wp_F}{R(x)(\varpi^4 - \sigma_{bj}^4)}$$

$$\left[\frac{G_F}{\sigma_{bj}}(\varpi^4 - \sigma_{bj}^4)(1 - cos\sigma_{bj})\right.$$

$$+(\varpi^2 - \sigma_{bj}^2)(cos\sigma_{bj}^2 t - cos\varpi\mathrm{t}$$

$$-A_m \frac{\varpi \mathrm{sin}\sigma_{bj}}{\sigma_{bj}} + A_m sin\varpi\mathrm{t}) - (\varpi^2 - \sigma_{bj}^2)$$

$$(cosh\varpi\mathrm{t} - B_m cos\sigma_{bj}\mathrm{t} - C_m sinh\varpi\mathrm{t}$$

$$\left.-C_m \frac{1}{\sigma_{bj}} sin\sigma_{bj}\mathrm{t})\right]$$

$$\left[sin\frac{\lambda_m x}{\mathrm{L}} + A_m cos\frac{\lambda_m x}{\mathrm{L}} + B_m sinh\frac{\lambda_m x}{\mathrm{L}} + C_m cosh\frac{\lambda_m x}{\mathrm{L}}\right] \quad (11)$$

Where $\wp_F = \frac{P}{\bar{m}\lambda_m[1-\varepsilon_0 LB_B(m,m)]}$ (12)

Practical Illustrations

Two classical boundary conditions are considered to demonstrate the general solution.

Clamped-Clamped Beam

For a beam clamped at both ends (x = 0 and x = L), the boundary conditions are:

$$\mathrm{V}(0,\mathrm{t}) = 0, \mathrm{V}(L,\mathrm{t}) = 0, \frac{\partial \mathrm{V}(0,\mathrm{t})}{\partial \mathrm{x}} = 0, \frac{\partial \mathrm{V}(\mathrm{L},\mathrm{t})}{\partial \mathrm{x}} = 0 \quad (13)$$

$$A_{\mathrm{m}} = \frac{\mathrm{sinh}\lambda_{\mathrm{m}} - \mathrm{sin}\lambda_{\mathrm{m}}}{cos\lambda_{\mathrm{m}} - cosh\ _{\mathrm{m}}} = \frac{cos\lambda_{\mathrm{m}} - cosh\lambda_{\mathrm{m}}}{\mathrm{sin}\lambda_{\mathrm{m}} - \mathrm{sinh}\ _{\mathrm{m}}}$$

$$-C_{\mathrm{m}} \text{ and } B_{\mathrm{m}} = -1 \quad (14)$$

and the frequency equation $cos\lambda_{\mathrm{m}} cosh\lambda_m = 1$. The first three eigenvalues are

$\lambda_1 = 4.73004$ A $\lambda_2 = 7.85320$

$\lambda_3 = 10.99561$ (Oni & Ogunyebi, 2008) (15)

Clamped-Free Beam

For beam clamped at x= 0 and free at $x =$ L:

$$\mathrm{V}(0,\mathrm{t}) = 0, \frac{\partial V(0,\mathrm{t})}{\partial x} = 0, \frac{\partial^2 \mathrm{V}(\mathrm{Ł},\mathrm{t})}{\partial x^2} = 0, \frac{\partial^3 \mathrm{V}(\mathrm{Ł},\mathrm{t})}{\partial x^3} 0 \quad (16)$$

The mode shape coefficient satisfies:

$$A_{\mathrm{m}} = \frac{\mathrm{sinh}\lambda_{\mathrm{m}} - \mathrm{sin}\lambda_{\mathrm{m}}}{cos\lambda_{\mathrm{m}} - cosh\lambda_{\mathrm{m}}} = \frac{cos\lambda_{\mathrm{m}} - cosh\lambda_{\mathrm{m}}}{\mathrm{sin}\lambda_{\mathrm{m}} - \mathrm{sinh}\lambda_{\mathrm{m}}}$$

$$=-C_{\mathrm{m}} \text{ and } B_{\mathrm{m}} = -1 \quad (17)$$

and the frequency equation $cos\lambda_{\mathrm{m}} cosh\lambda_{\mathrm{m}} = -1$.The first three eigenvalues are

$\lambda_1 = 1.875$ A $\lambda_2 = 4.694$ $\lambda_3 = 7.855$ (18)
(Oni & Ogunyebi, 2008)

Resonance Discussion

For the moving force solution, resonance occurs when:

$$\dot{\omega}_{bj} = \frac{\lambda_m u}{\mathrm{L}} \quad (19)$$

For the moving mass, resonance conditions are modified to:

$$\sigma_{bj} = \frac{\lambda_{\mathrm{m}} \mathrm{u}}{\mathrm{L}} \quad (20)$$

where

$$\sigma_{bj} = \omega_{b\,j}\left\{1 - \varepsilon_1\left[\frac{v^2 \mathrm{ŁB}(\mathrm{m},\mathrm{m})}{8}_1 + \frac{ŁD_{A1}(t)}{2} - \left(\frac{\omega_{b\,j}{}^2 ŁB_C(\mathrm{m},\mathrm{m})}{2} + \frac{\omega_{b\,j} ŁD_{A1}(t)}{2}\right)\right]\right\} \quad (21)$$

This shift indicates that the moving mass reaches resonance at lower critical speed than the moving force.

Numerical Simulation and Discussions

Numerical simulations were performed the uniform beam with length L = 12.192m, Young's modulus E =$3.1\mathrm{x}10^{10}$ N/m^2 and the moment of inertia L = $2.87698\mathrm{x}10^{-3}$m^4, mass per unit length $\overline{m} = 2758.291 kg/m$, mass ratio $\varepsilon = 0.25$ and load velocity u = 8.128m/s. The values of C and C_s were varied between 0 and 10 while other parameters were at fixed K =40000, N=20000, G=10000, R^0 = 10 (Famuagun, 2023) . The results are shown below.

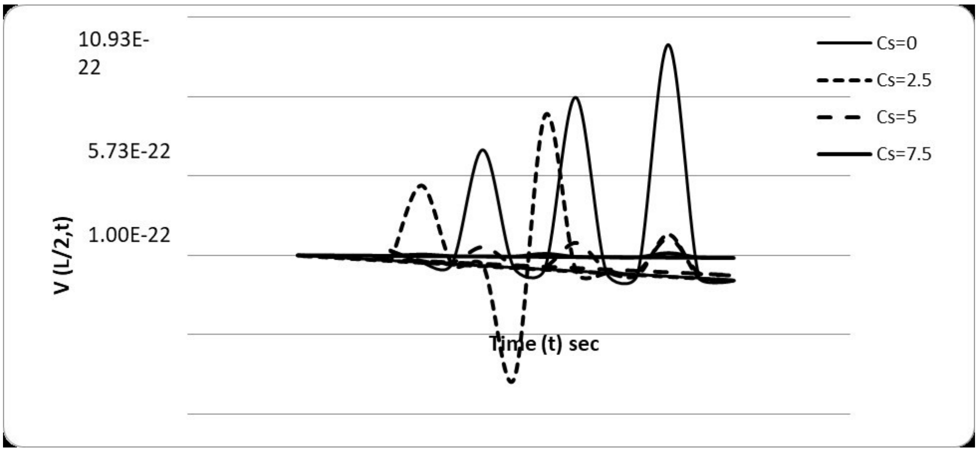

Figure 1: Displacement response of damped structure clamped at both ends under the influence of moving force for various values of Cs.

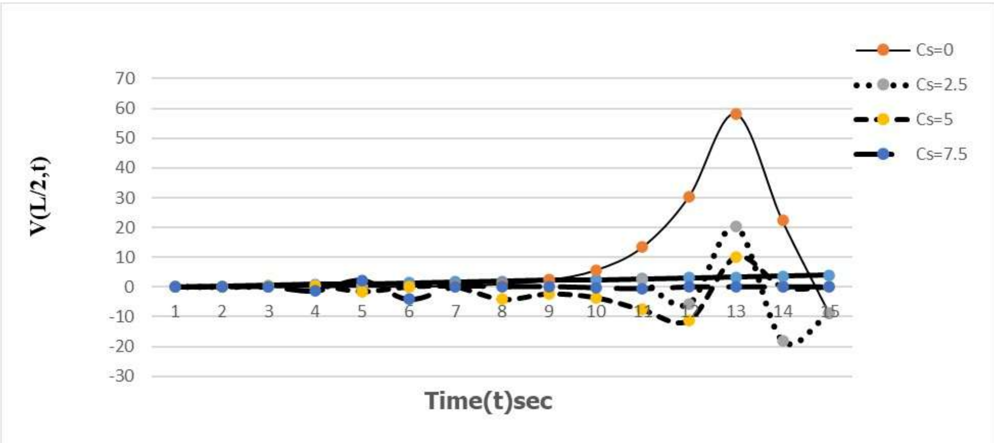

Figure 2: Displacement response of damped structure clamped at both ends under the influence of moving mass for various values of C_s.

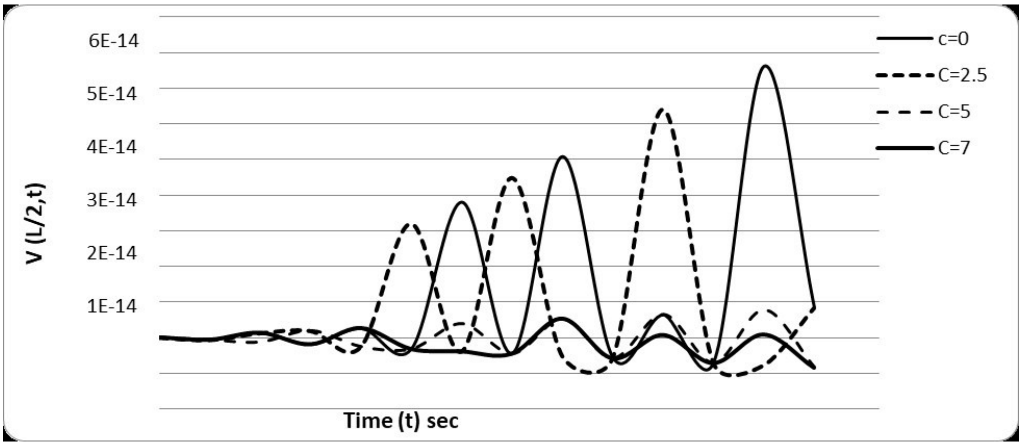

Figure 3: Displacement response of damped structure clamped at both ends under the influence of moving force for various values of C.

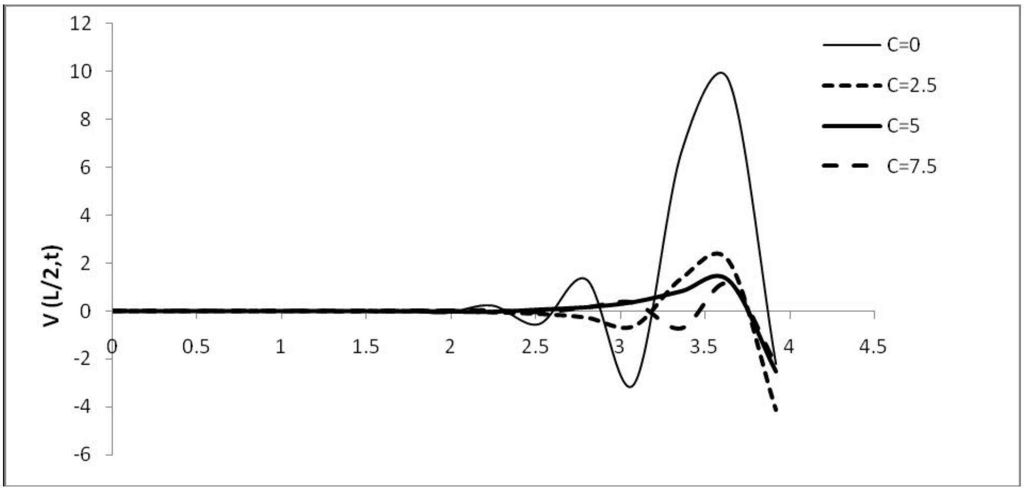

Figure 4: Displacement response of damped structure clamped at both ends under the influence of moving mass for various values of C.

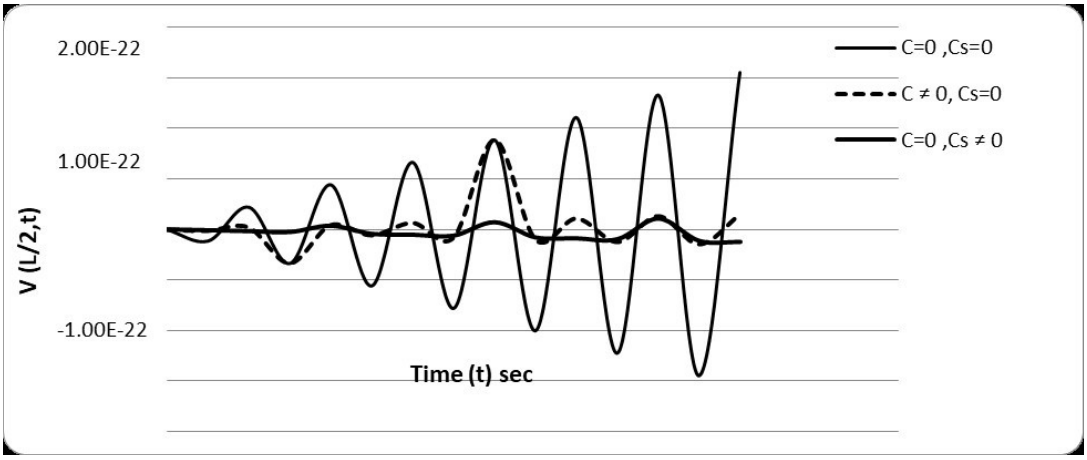

Figure 5: Comparison of effect of the two damping parameters on the uniform structure clamped at both ends and subjected to moving forces.

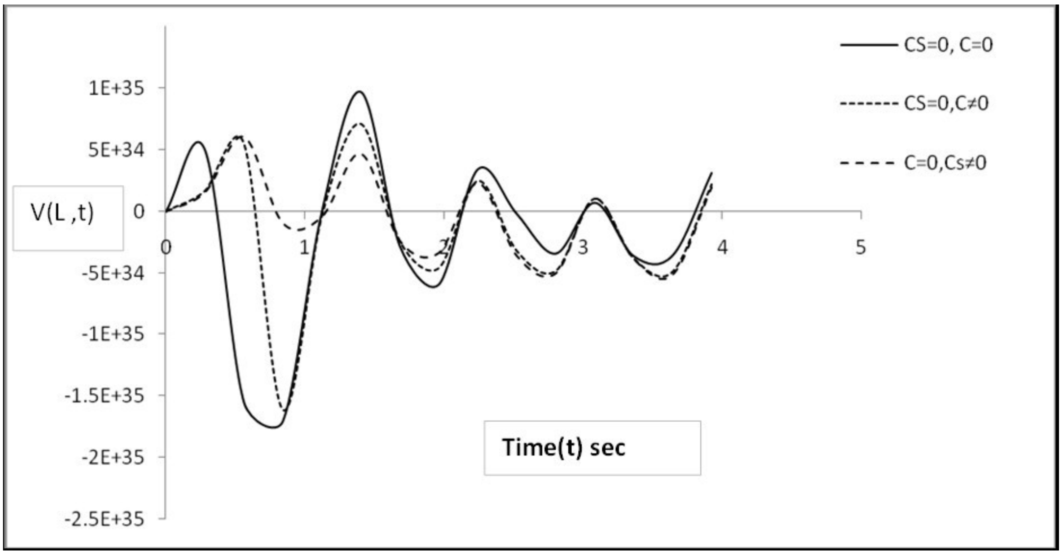

Figure 6: Comparison of the effects of the two damping parameters on the uniform structure clamped at both ends and subjected to moving masses.

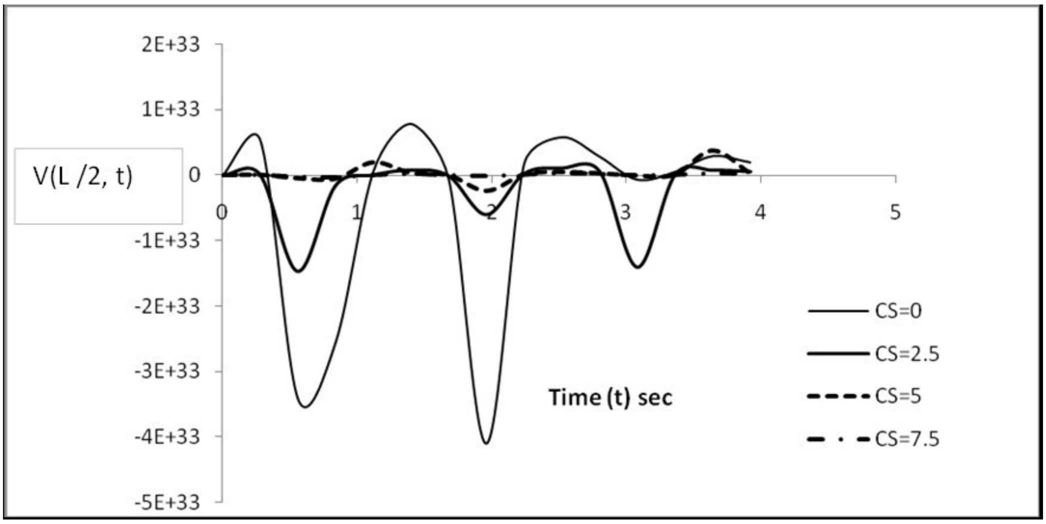

Figure 7: Deflection profile of damped structure under the influence of moving force clamped at one end and free other end for various values of C_s

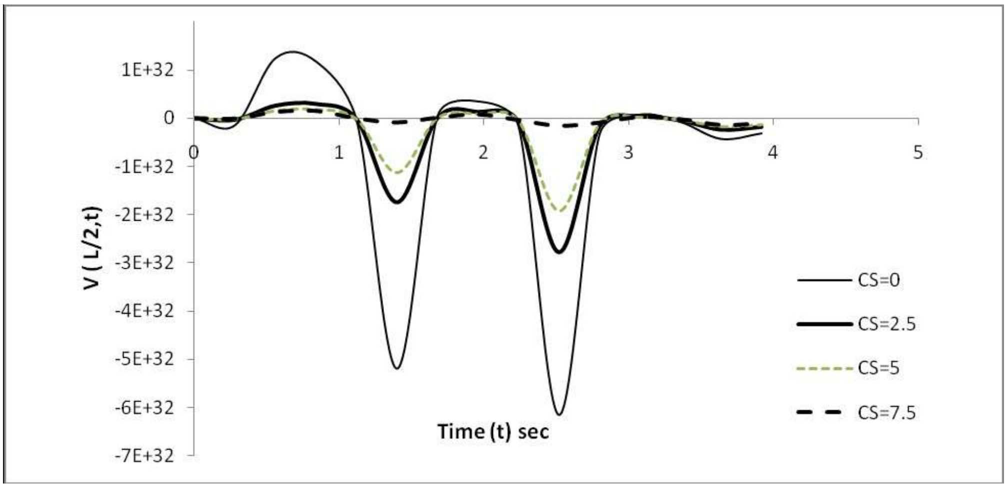

Figure 8: Deflection profile of moving distributed masses for clamped-free damped beam with various values of C_s

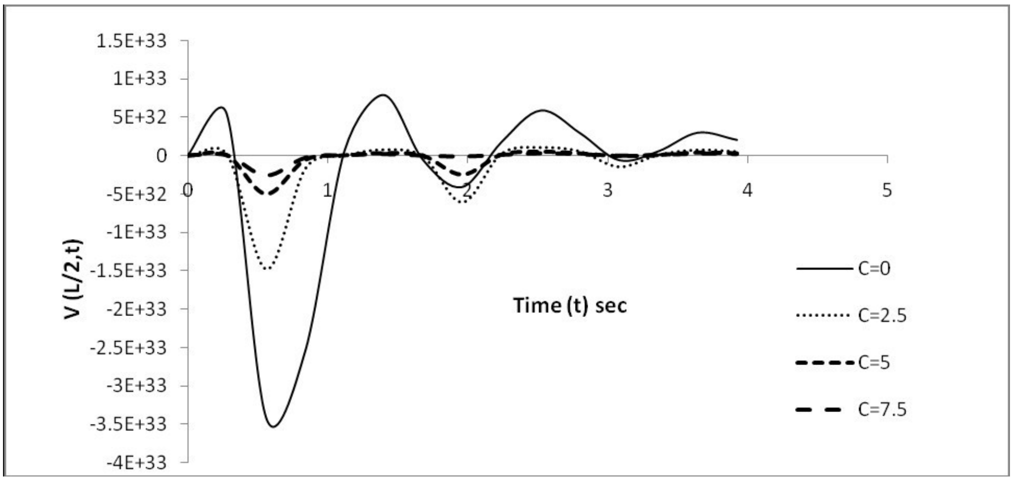

Figure 9: Deflection profile of damped structure under the influence of moving force clamped at one end and free other end with various values of C.

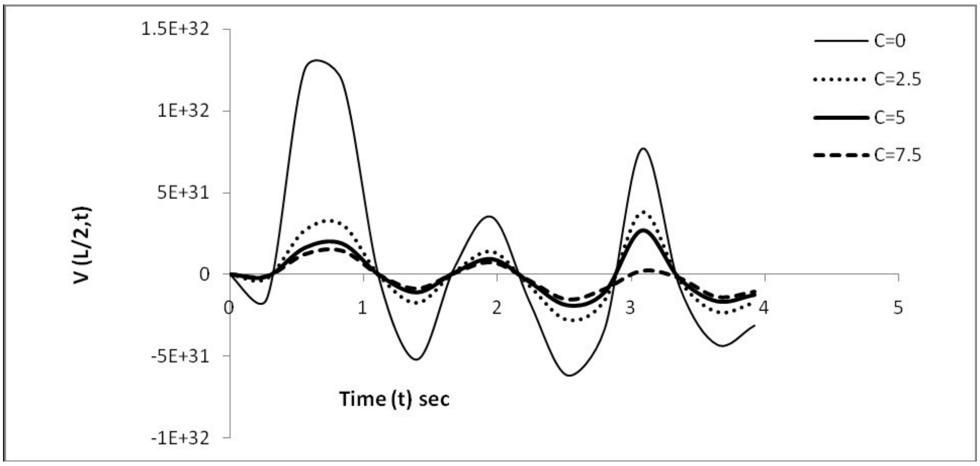

Figure 10: Deflection profile of damped structure under the influence of moving mass clamped at one end and free at one end for various values of C.

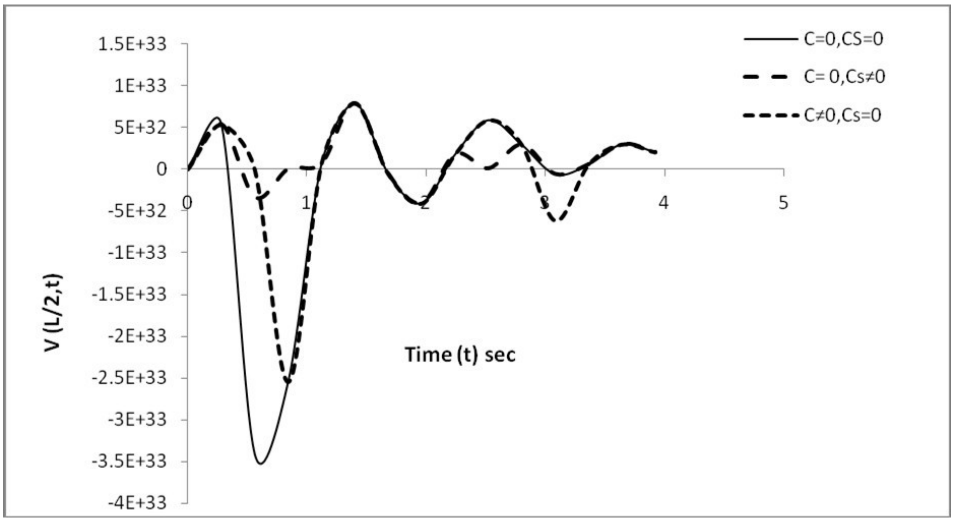

Figure 11: Comparison of the effects of the two damping parameters for clamped-free uniform structure subjected to moving forces.

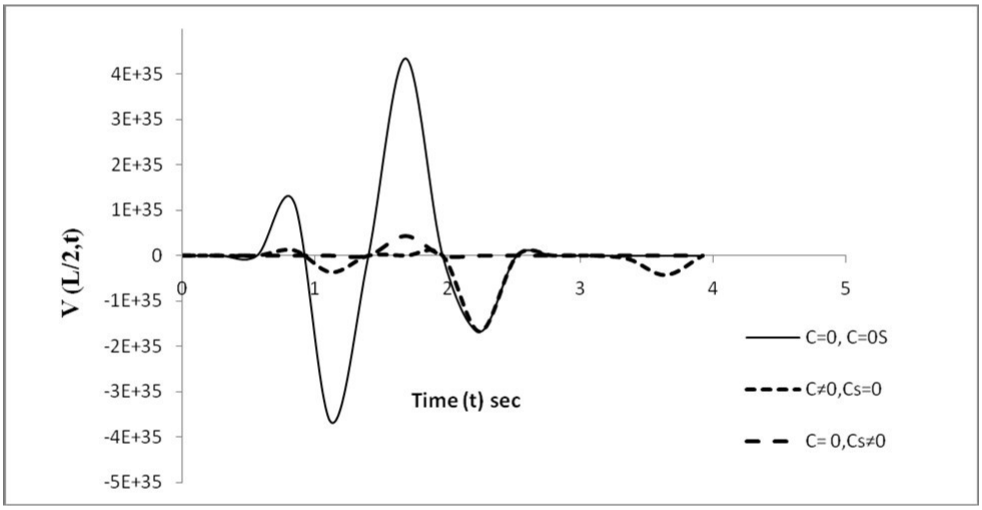

Figure 12: Comparison of the effects of the two damping parameters for clamped-free uniform structure subjected to moving masses.

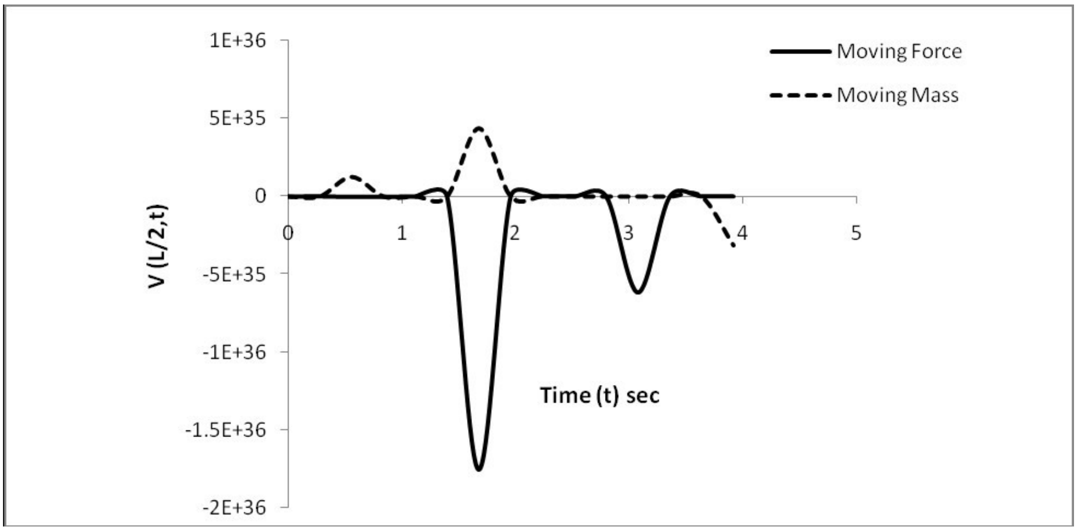

Figure 13: Comparison of deflection of moving force and moving mass cases of a uniform beam at constant velocity.

Figures 1 and 2 (for clamped-clamped) and 7 and 8 (for clamped-free) show that increasing Cs significantly reduces the vibration amplitude for both moving forces and moving masses. A similar trend is observed for increasing C in Figure 3, 4, 9 and 10, though the reduction is less pronounced. This is further illustrated in Figure 5 - 6 and 11-12, which directly compare the two damping effects: Cs is more effective in suppressing beam deflection than C under both load types.

Figure 13 compares the displacement response for the moving force and moving mass cases. At the same constant velocity and fixed structural parameters, the moving force produces a larger amplitude response than the moving mass. This implies that the moving force solution does not serve as an upper bound for the moving mass problem; instead, the inclusion of load inertia alters the effective stiffness and modifies the critical speed.

Conclusion

This study investigated damping effects on the dynamic behaviour of a beam-type structure with classical boundary condition under moving distributed load. Analytical approach was developed to solve the governing equation.

The following are the main results obtained from the study:

(i) The vibration amplitude is significantly reduced by both damping coefficients.

(ii) The effect of damping due to strain velocity is observed to be less effective to damping due to transverse displacement Cs.

(iii) Resonance occurs earlier in moving mass case than in the moving force case.

(iv) Moving force responses are generally larger than moving mass responses.

Thus, to mitigate vibration effects and prevent structural failure, the impact of incorporating appropriate damping mechanisms in structural design cannot be over-emphasized. The results agreed with Idowu, *et al* (2008) and Titiloye *et al* (2014).

References

Adams G. G. (1995): Critical speeds and response of a tensioned beam on an elastic foundation to repetitive moving loads: J. Mech. Science, Vol. 37, No 7 pp 773-781. (1)

Crandall, S. H. (1970). The role of damping in vibration theory. *Journal of Sound and Vibration, 11*(1), 3-18. https://doi.org/10.1016/S0022-460X(85)81214-1

Famuagun K. S. (2023) Influence of damping coefficients on the response to moving distributed Masses of Rayleigh Beams resting on biparametric subgrade. Journal of the Nigerian Association of Mathematical Physics. Vol 65, pp 71-86.

Idowu, A.S., Titiloye, E.O., Dada, M.S. and Gbadeyan J. A. (2008): The effect of Viscons damping on the dynamic behaviour of rectangular plates resting on elastic foundation under moving loads: Jour. of inst. Of maths. And comp. Sciences. Vol. 21, No 2, 117-123.

Lan Ding, Hong-Ping Zhu and Li Wu (2016): Effect of axial load and structural damping on wave propagation in periodic Timoshenko beams on elastic foundation under moving load. Eaculty of Engineering, China Union of Geoscience, Wuhan 430074 PR China.School of Engineering and Mechanic, Huozhong Union of science and Technology, Wuhan 430074 PR China.VOL380, Issue 31-32.233-532.

Li, P., Wang, T., & Zhao, R. (2024). Analytical investigation of damping effects in beam-type structures under moving inertial loads. Applied Mathematical Modelling, 125, 455–472.

Oliveto G. and Greco A. (2002): Some observations on the characterization of structural damping. Journal of sound and vibration, 256(3), 391-415. Available online at http//www.idealibrary.com.

Oni S. T. and Ogunyebi S. N. (2008). Dynamical nalysis of a prestress beam with general boundary conditions under the action of uniform distributed masses. Journal of the Nigerian Association of Mathematical Physics. Vol 12, 87-102.

Oni, S.T. and Omolofe, B. (2011): Dynamic response of prestressed Rayleigh beam resting on elastic foundation and subjected to masses travelling at varying velocity, Journal of Vibration and Acoustics (USA). Vol. 133, pp 041005-1-15.

Titiloye E. O., Famuagun, K. S. and Gbadeyan J. A. (2014). The effect of damping on the dynamic response of beam subjected to distributed Load. Journal of the Nigerian Association of Mathematical Physics (NAMP), 26, 99-108.

Timoshenko, S. (1992). On the transverse vibration of bars of uniform cross-section. Philosophy Mag. Series 6, Vol. 43, pp 125-131.

Wu, Y. and Gao, Y. (2015): Analytical solutions for simply supported viscously damped double-beam system under moving harmonic loads. Journal of Engineering Mechanics for American Society of Civil Engineers. Vol. 141, No. 7.

Zhang, Y., Liu, H., & Chen, X. (2023). Vibration suppression in beam structures subjected to moving masses using viscous damping mechanisms. Journal of Sound and Vibration, 547, 117498.

Zhao, X., Zhu, W. D., & Li, Y. H. (2023). Effects of strain-rate-dependent damping on dynamic response of beams under high-speed moving loads. International Journal of Mechanical Sciences, 240, 107922.